

# **Smith & Dewar**

## **SAT/PSAT\* PREP Materials**

### **Geometry Section**

by **Sharon Smith and Walter Dewar**



**Presented by Advanced Placement\* Strategies**

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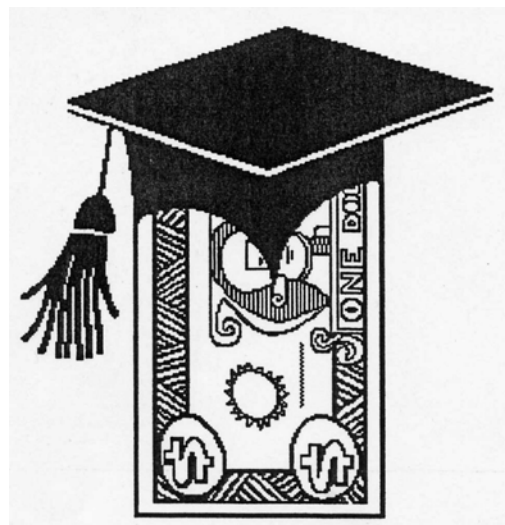
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# Geometry



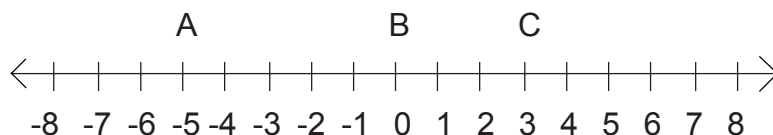
This geometry section is a collection twelve sets of geometry problems that are similar to those found on SAT/PSAT tests.

1. The number line
  2. Linear pairs, vertical angles, and perpendicular lines
  3. Triangle sum theorem and parallel lines
  4. Exterior angles of triangles
  5. Isosceles triangles
  6. Quadrilaterals
  7. Ratio and proportion
  8. The Pythagorean theorem and special right triangle ratios
  9. Circles
  10. Area
  11. The coordinate plane
  12. Volumes and 3-dimensional shapes
- When a geometry teacher gives a test that covers any of these concepts, we recommend that the teacher distribute and assign the corresponding section of prep problems and that the teacher go over the answers on the day following the test.

# #1

## The number line

When numbers are assigned to points on a line, it is called a number line.



The number corresponding to A, known as its coordinate, is -5.

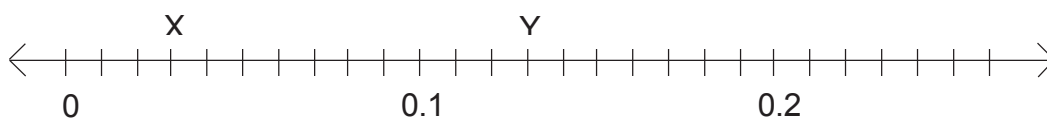
The coordinate of B is 0.

The coordinate of C is 3.

AC means the distance from A to C or C to A, and it is 8.  
AB means the distance from A to B or B to A, and it is 5. } Distance is always a positive number.

Do the following PSAT/SAT style problems involving the number line.

1. In the given figure, what is the length of  $\overline{XY}$ ?

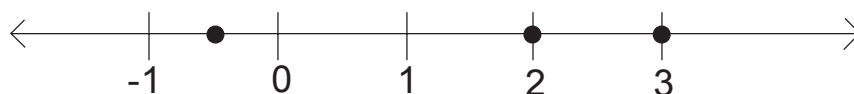


(A) 10 (B) 1.0 (C) 0.1 (D) 1.3 (E) 0.13

2. What is the closest approximation of the product of the coordinates of the three points on the given number line?

Know the following:

- 1) product is the result of a multiplication problem
- 2) sum is the result of an addition problem
- 3) difference is the result of a subtraction problem
- 4) quotient is the result of a division problem



(A) -3 (B) -1 (C) 0 (D) 1/6 (E) 1

3. On a number line, if point A is a distance of 3 from point X and point B is a distance of 5 from point X, what is the shortest possible distance from A to B?

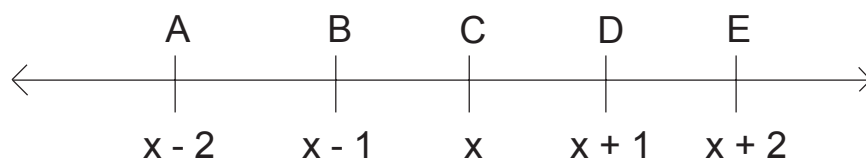
(A) 0 (B) 2 (C) 3 (D) 8 (E) 15

4. For the given number line, if  $AB > BC > CD > DE$ , which of the following must be true?



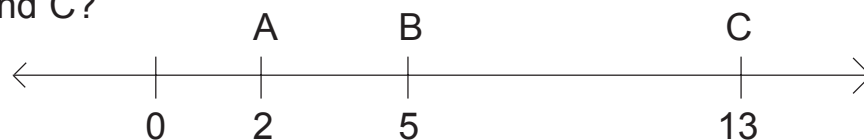
- (A)  $BC > CE$  (B)  $AC > BE$  (C)  $BE > AB$  (D)  $AC > CE$  (E)  $AB > BD$

5. The fraction  $\frac{4x-1}{4}$  falls between which two points on the given number line?



- (A) A and B (B) B and C (C) C and D (D) D and E  
(E) none of the above

6. If A, B and C represent numbers on the given number line and  $AB < BC$ , then which of the following represents the position of the point whose coordinate is equal to the average of the coordinates of A, B and C?



- (A) Between 0 and A (B) Between A and B (C) Between B and C  
(D) Same as point B (E) To the right of C

See the top of the next page if you would like a hint for the next question, #7.

7. If two points, A and B, are each placed to the right of point P on the given line such that  $3PA = 4PB$ , what is  $AB : PB$ ?



- (A) 1:3 (B) 1:2 (C) 1:4 (D) 2:3 (E) 3:4

Hint for #7: Some values that will make an equation such as  $2x = 5y$  a true statement are  $x = 5$  and  $y = 2$ .  $(2)(5) = (5)(2)$ .  
 Another set is  $x = 10$  and  $y = 4$ .  $(2)(10) = (5)(4)$ .  
 For #7, if  $3(PA) = 4(PB)$ , assign values for  $PA$  and  $PB$  that will make  $3PA = 4PB$ , and use your values for  $PA$  and  $PB$  to find  $AB$ .

The following are student-produced response questions. You will enter your answer by marking the ovals in a grid like the one below.

8. If  $AB = 52$  and  $BC = 38$ , what is the distance from  $A$  to the midpoint of  $BC$ ?



9.  $B$  is the midpoint of line segment  $AC$ ,  $X$  is the midpoint of line segment  $AB$ , and  $Y$  is the midpoint of line segment  $BC$ . If the length of  $XY$  is 10, what is the length of  $AC$ ?



10. On a number line, if point  $X$  has a coordinate of  $-3$  and point  $Y$  has a coordinate of  $12$ , what is the coordinate of the point that is located  $1/3$  of the way from  $X$  to  $Y$ ?

11. On a number line, if point  $X$  has a coordinate of  $-4$  and point  $Y$  has a coordinate of  $16$ , what is the coordinate of the point that is located  $3/4$  of the way from  $X$  to  $Y$ ?

12. What is the coordinate of the point on the number line that is exactly half way between  $17$  and  $26$ ?

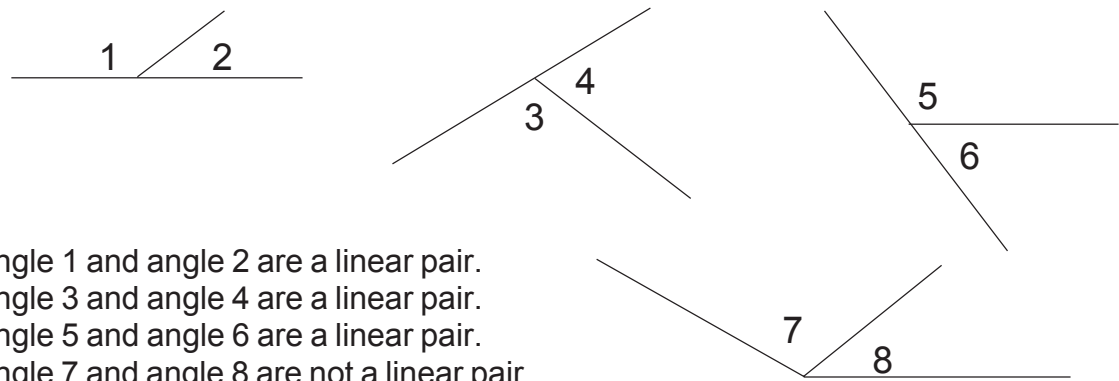
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3	3	3	3	3
4	4	4	4	4
5	5	5	5	5
6	6	6	6	6
7	7	<input checked="" type="radio"/>	7	7
8	8	8	8	8
9	9	9	9	9

Answers: 1. C 2. A 3. B 4. D 5. B 6. C 7. A 8. 71 9. 20 10. 2 11. 11 12. 21.5

# #2

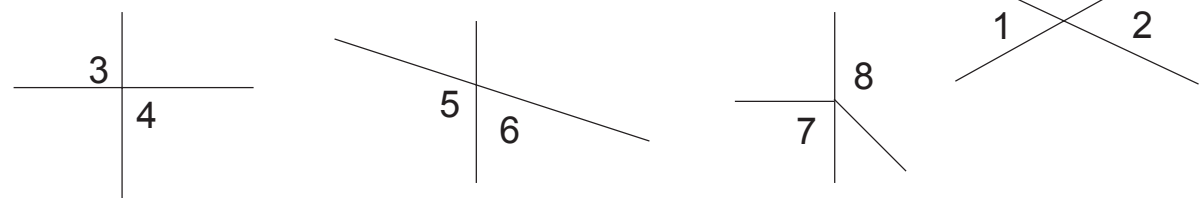
## Linear pairs, vertical angles and perpendicular lines

Two angles that are adjacent and supplementary are called a linear pair. Two of the rays form a straight line. The measures of the two angles add up to  $180^\circ$ .



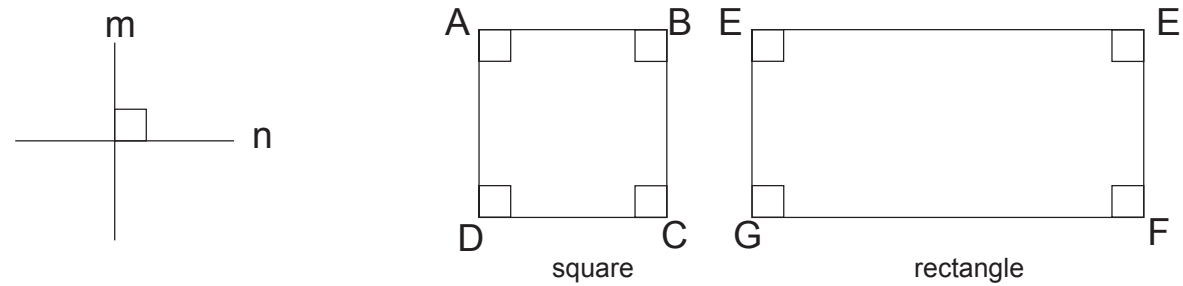
Angle 1 and angle 2 are a linear pair.  
 Angle 3 and angle 4 are a linear pair.  
 Angle 5 and angle 6 are a linear pair.  
 Angle 7 and angle 8 are not a linear pair.

Vertical angles are the nonadjacent angles formed by two intersecting lines. Vertical angles have the same measure and are congruent.



Angle 1 and angle 2 are vertical angles.  
 Angle 3 and angle 4 are vertical angles.  
 Angle 5 and angle 6 are not vertical angles. These angles are adjacent.  
 Angle 7 and angle 8 are not vertical angles. There are not two intersecting lines. Calling angles like 7 and 8 vertical angles is a common mistake made by geometry students.

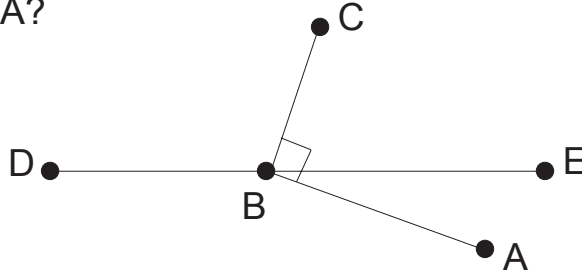
Two lines are perpendicular if they intersect to form a right angle.



Any two adjacent sides of a square or rectangle are perpendicular.  $\overline{AB} \perp \overline{BC}$ .

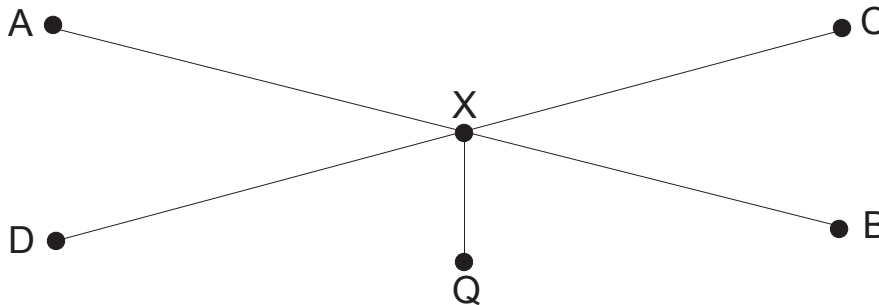
The answers are at the bottom of the last page.

1. In the given figure,  $\overline{AB} \perp \overline{BC}$ , and the  $m\angle DBC = 110^\circ$ . What is  $m\angle EBA$ ?



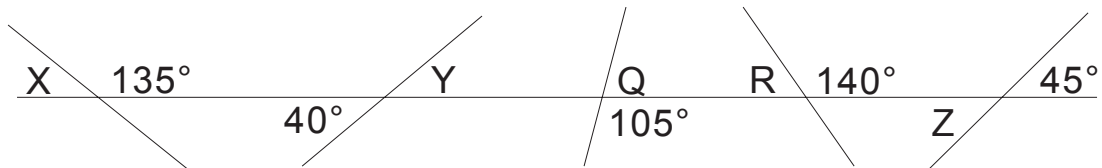
- (A)  $20^\circ$  (B)  $50^\circ$  (C)  $70^\circ$  (D)  $90^\circ$  (E)  $110^\circ$

2. In the given figure,  $\overline{AB}$  and  $\overline{CD}$  intersect at X. If  $m\angle AXC = 110^\circ$  and  $m\angle DXQ = m\angle BXQ$ , what is  $m\angle DXQ$ ?



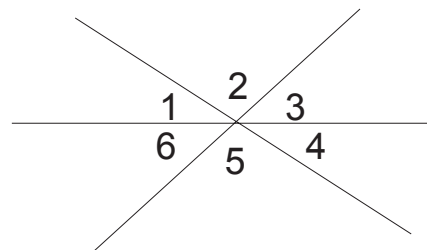
- (A)  $45^\circ$  (B)  $55^\circ$  (C)  $65^\circ$  (D)  $110^\circ$  (E)  $220^\circ$

3. In the given figure, which angle has the same measure as X?



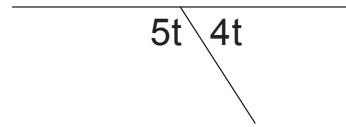
- (A) Y (B) Q (C) R (D) Z (E) None of these angles

4. In the given figure, all three line segments intersect at the same point. The  $m\angle 1 + m\angle 3$  must be equal to which of the following?



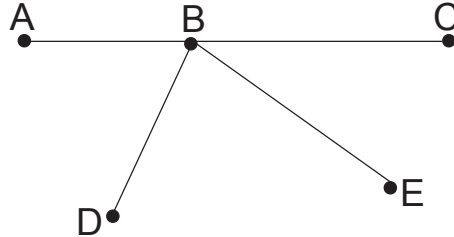
- (A)  $m\angle 4 + m\angle 5$  (B)  $m\angle 2 + m\angle 5$   
 (C)  $m\angle 1 + m\angle 4$  (D)  $m\angle 3 + m\angle 6$  (E)  $m\angle 4 + m\angle 6$

5. In the given figure, what is the value of  $t$ ?



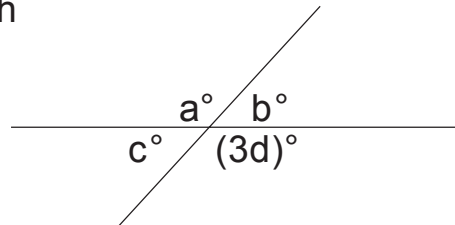
- (A)  $10^\circ$  (B)  $20^\circ$  (C)  $30^\circ$  (D)  $80^\circ$  (E)  $100^\circ$

6. In the given figure, if  $m\angle ABE = 138^\circ$  and  $m\angle CBD = 119^\circ$ , find  $m\angle DBE$ .



- (A)  $76^\circ$  (B)  $77^\circ$  (C)  $78^\circ$   
(D)  $79^\circ$  (E)  $80^\circ$

7. With respect to the given figure, which of the following must equal  $180 - d$ ?



- (A)  $a + 2d$  (B)  $2b + d$  (C)  $c + 2d$   
(D)  $a + 2c$  (e)  $c + 2a$

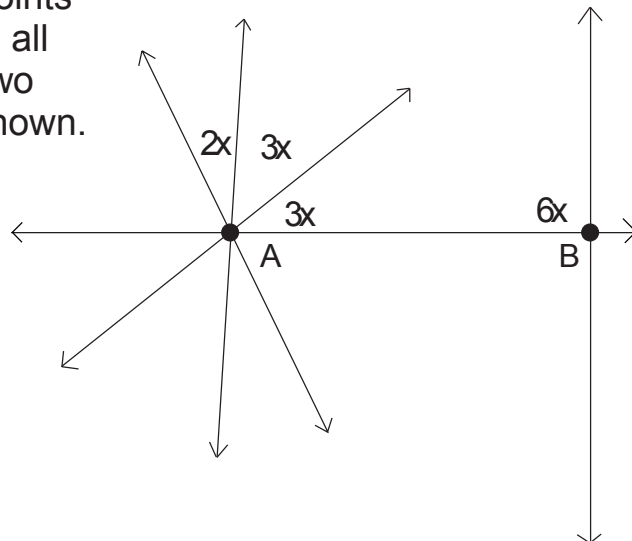
A method for solving many math problems with variables is this:

**Whenever you see a variable, consider substituting in a value.**

Begin by choosing a value for  $d$  such as 40. If  $d = 40$ , then  $b = 60$ ,  $a = 120$  and  $c = 60$ . Second, answer the question  $(180 - d)$  using your values. You will get  $180 - 40 = 140$ . Third, insert your values for the variables into each of the answer choices and see which one matches your answer of **140**. Choice (C) gives **140**.

$$60 + 2(40) = 60 + 80 = 140$$

8. In this figure, which is not drawn to scale,  $x = 15^\circ$ . How many total points of intersection will there be when all lines are extended indefinitely? Two intersection points, A and B, are shown.

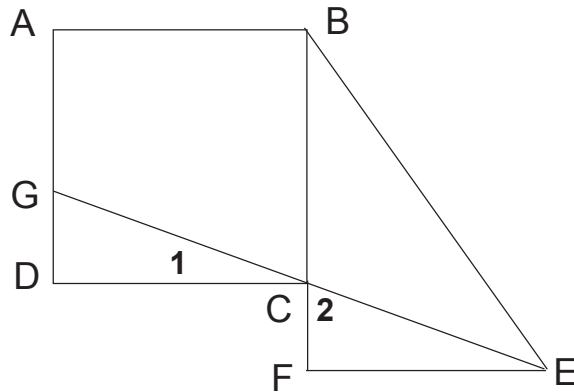


- (A) 2 (B) 3 (C) 4  
(D) 5 (E) 6

Hint: Redraw the diagram to scale.

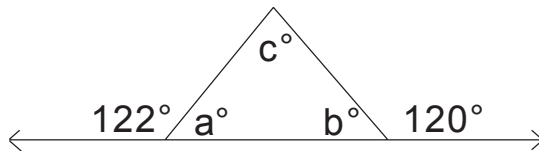
The following are student-produced response questions. You will enter your answer by marking the ovals in a grid like the one below.

9. Quadrilateral ABCD is a square. BEF is a triangle. GE is a line segment passing through C. Find  $m\angle 1 + m\angle 2$ .

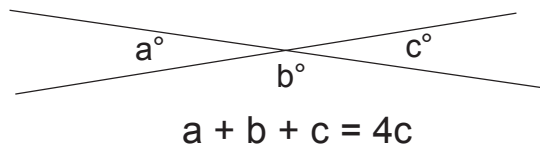


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	0	0	<input checked="" type="radio"/>
1	1	1	1
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4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	<input checked="" type="radio"/>	9

10. In the given figure, what is the value of  $c$ ?



11. In the given figure,  $a + b + c = 4c$ . What is the value of  $c$ ?



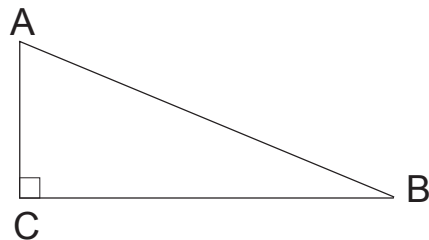
Answers: 1. A 2. B 3. D 4. E 5. B 6. B 7. C 8. C 9. 90 10. 62 11. 60

# #3

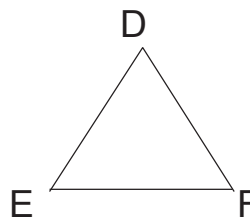
## Triangle sum theorem and parallel lines

The sum of the measures of the angles of a triangle is  $180^\circ$ .

If  $m\angle A = 60^\circ$  and  $m\angle C = 90^\circ$ , then  $m\angle B = 30^\circ$ .



If  $\triangle DEF$  is an equilateral triangle, then  $m\angle D = m\angle E = m\angle F = 60^\circ$ .



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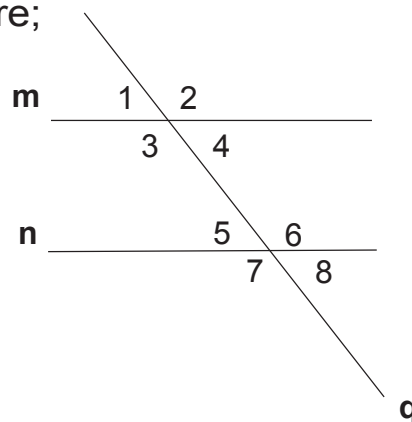
**If two parallel lines (m and n) are cut by a transversal (q), then**

1) alternate interior angles (3 & 6, 4 & 5) are congruent ( $\cong$ ) and have the same measure;

2) corresponding angles (1 & 5, 2 & 6, 3 & 7, and 4 & 8) are congruent and have the same measure;

3) alternate exterior angles (1 & 8, 2 & 7) are congruent and have the same measure;

4) interior angles on the same side of the transversal (3 & 5, 4 & 6) are supplementary and their measures have a sum of  $180^\circ$ .



If  $m\angle 1 = 60^\circ$ , then

$$m\angle 5 = 60^\circ$$

$$m\angle 4 = 60^\circ$$

$$m\angle 8 = 60^\circ$$

$$m\angle 6 = 120^\circ$$

$$m\angle 2 = 120^\circ$$

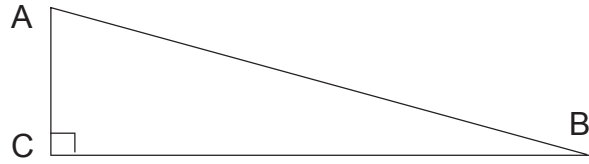
$$m\angle 7 = 120^\circ$$

$$m\angle 3 = 120^\circ$$

The answers are at the bottom of the last page.

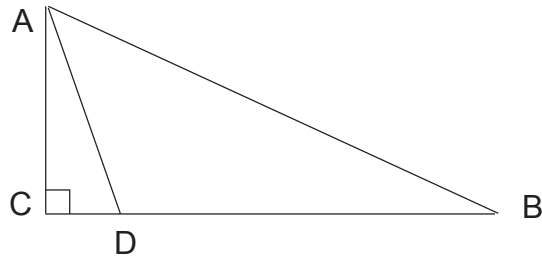
1. If  $m\angle B = 20^\circ$ , what is  $m\angle A - m\angle B$ ?

- (A)  $40^\circ$  (B)  $50^\circ$  (C)  $60^\circ$   
 (D)  $70^\circ$  (E)  $90^\circ$



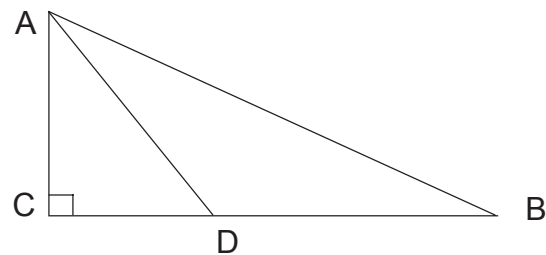
2. Which of the following statements must be true?

- (A)  $m\angle DAB > m\angle B > m\angle CAB$   
 (B)  $m\angle ADC > m\angle C > m\angle B$   
 (C)  $m\angle B > m\angle CAB > m\angle DAB$   
 (D)  $m\angle C > m\angle CAB + m\angle B$   
 (E)  $m\angle ADB > m\angle C > m\angle B$



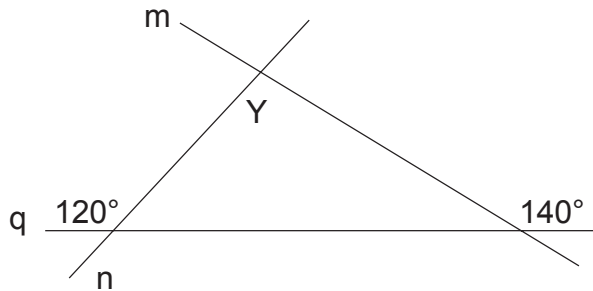
3. If  $\overline{AD}$  is the bisector of  $\angle CAB$  and  $m\angle ADB = 110^\circ$ , what is  $m\angle B$ ?

- (A)  $20^\circ$  (B)  $30^\circ$  (C)  $40^\circ$   
 (D)  $50^\circ$  (E)  $60^\circ$



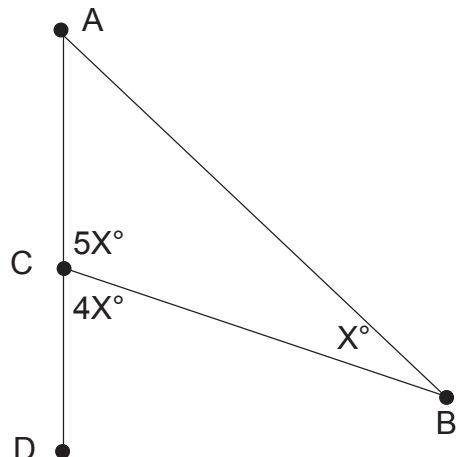
4. If lines  $m$ ,  $n$  and  $q$  intersect as shown in the given diagram, what is the value of  $Y$ ?

- (A)  $40^\circ$  (B)  $60^\circ$  (C)  $80^\circ$   
 (D)  $90^\circ$  (E)  $100^\circ$



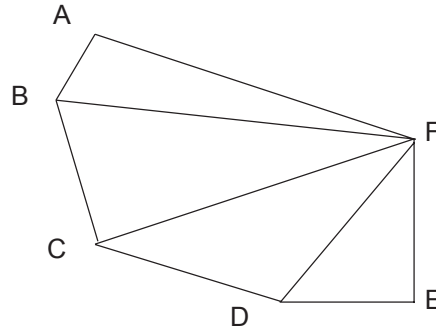
5. In the given figure, what is  $m\angle A$ ?

- (A)  $20^\circ$   
 (B)  $30^\circ$   
 (C)  $40^\circ$   
 (D)  $50^\circ$   
 (E)  $60^\circ$



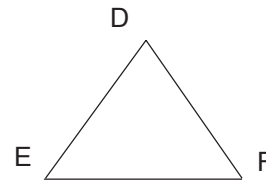
6. What is the sum of  $m\angle A + m\angle ABC + m\angle BCD + m\angle CDE + m\angle E + m\angle EFA$ ?

- (A)  $180^\circ$  (B)  $360^\circ$  (C)  $540^\circ$   
 (D)  $720^\circ$  (E)  $900^\circ$



7. In the given diagram,  $m\angle E = 61^\circ$  and  $m\angle F = 59^\circ$ . What is  $m\angle D$ ?

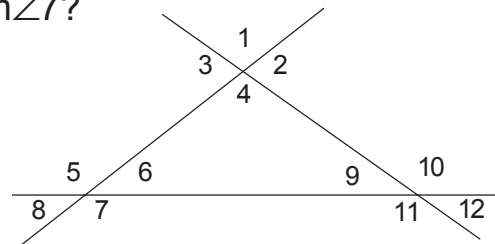
- (A)  $58^\circ$  (B)  $59^\circ$  (C)  $60^\circ$  (D)  $61^\circ$  (E)  $62^\circ$



8. In the given figure, what is the sum of  $m\angle 3, m\angle 2, m\angle 10, m\angle 11, m\angle 5$  and  $m\angle 7$ ?

Explanations below.

- (A)  $180^\circ$  (B)  $360^\circ$  (C)  $540^\circ$   
 (D)  $720^\circ$  (E)  $900^\circ$



Explanation: *Do not read this until you have tried the problem.*

First, recognize that  $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360^\circ$  and  
 $m\angle 5 + m\angle 6 + m\angle 7 + m\angle 8 = 360^\circ$  and  
 $m\angle 9 + m\angle 10 + m\angle 11 + m\angle 12 = 360^\circ$

The sum of the measures of all of these angles is  $1080^\circ$ .

Since the sum of the measures of angles 4, 6 and 9 is  $180^\circ$  (the triangle sum theorem), the sum of the three vertical angles to 4, 6 and 9 (angles 1, 8 and 12) is also  $180^\circ$ .

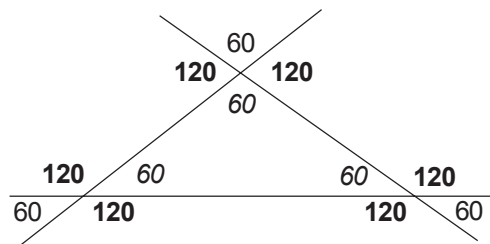
So, the measures of angles

$$(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12) - (4 + 6 + 9) - (1 + 8 + 12) = (2 + 3 + 5 + 7 + 10 + 11)$$

$$1080^\circ - 180^\circ - 180^\circ = 720^\circ$$

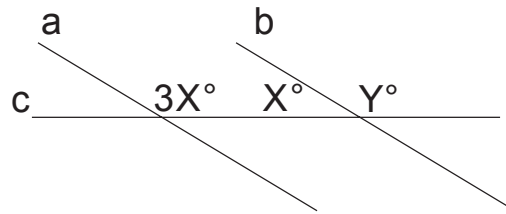
Another method: since  $m\angle 4 + m\angle 6 + m\angle 9 = 180^\circ$ , assign each of these angles a measure of  $60^\circ$ , and based on this, find the measures of the other angles. The diagram shows the result.

Since  $m\angle 3, m\angle 2, m\angle 10, m\angle 11, m\angle 5$  and  $m\angle 7$  each have a measure of  $120^\circ$ , the sum is  $720^\circ$ .



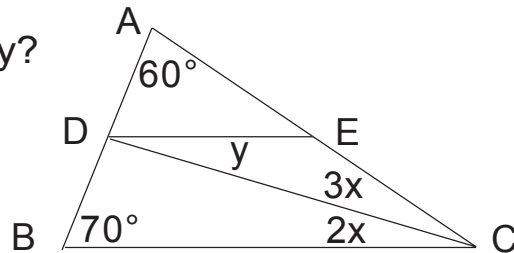
9. If  $a \parallel b$ , what is the value of  $Y$ ?

- (A)  $45^\circ$  (B)  $100^\circ$  (C)  $120^\circ$   
 (D)  $135^\circ$  (E)  $150^\circ$



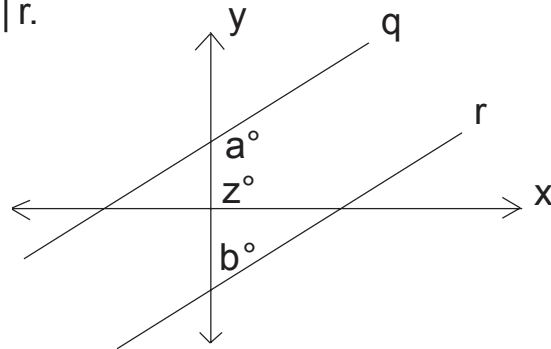
10. If  $\overline{DE} \parallel \overline{BC}$ , what is the value of  $y$ ?

- (A)  $10^\circ$  (B)  $20^\circ$  (C)  $30^\circ$   
 (D)  $40^\circ$  (E)  $50^\circ$



11. In the given coordinate plane,  $q \parallel r$ .  
 What is the sum of  $a$ ,  $b$  and  $z$ ?

- (A) 180  
 (B) 200  
 (C) 235  
 (D) 270  
 (E) 360



12. Given four distinct lines named  $l_1, l_2, l_3$  and  $l_4$ , which of the following are possible numbers of intersections if  $l_1 \parallel l_2$  and  $l_3 \parallel l_4$ ? All lines are in the same plane.

- I. zero  
 II. two  
 III. four

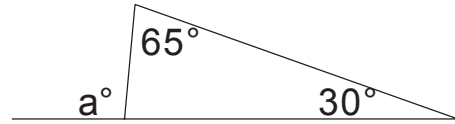
- (A) I only (B) I and II only (C) II and III only (D) III only  
 (E) I and III only

13. Which of the following is not a possibility for the number of points of intersection of three lines in the same plane?

- (A) 0 (B) 1 (C) 2 (D) 3  
 (E) All of the above are possible.

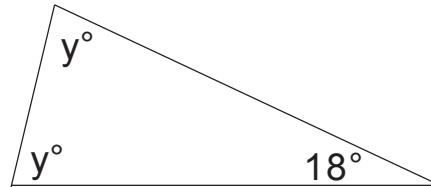
14. In the given diagram, what is the value of  $a$ ?

- (A) 85 (B) 90 (C) 95  
(D) 100 (E) 105



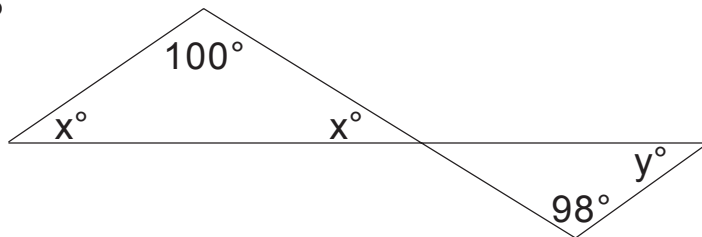
15. In the given diagram, what is the value of  $y$ ?

- (A) 50 (B) 61 (C) 71  
(D) 81 (E) 91



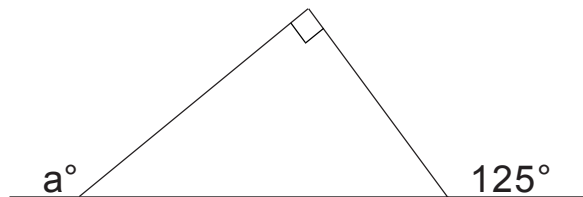
16. In the given diagram, what is the value of  $y$ ?

- (A) 51 (B) 32 (C) 38  
(D) 41 (E) 42



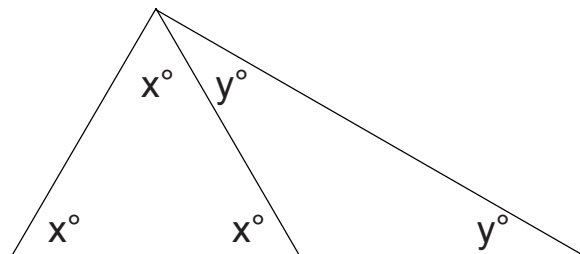
17. In the given diagram, what is the value of  $a$ ?

- (A) 115 (B) 125 (C) 135  
(D) 145 (E) 155



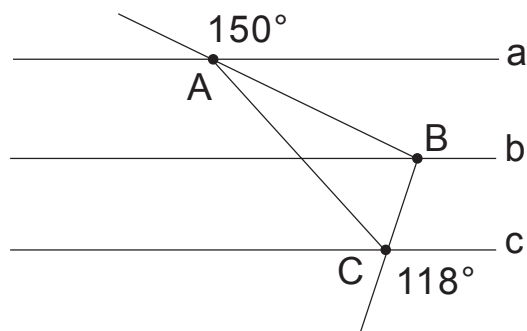
18. In the given diagram, what is the value of  $y$ ?

- (A) 20 (B) 25 (C) 30  
(D) 35 (E) 40



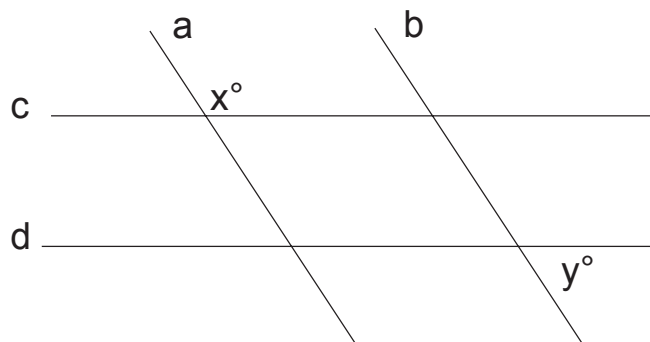
The following are student-produced response questions. You will enter your answer by marking the ovals in a grid like the one below.

19. In the figure below,  $a \parallel b \parallel c$ . If the measure of  $\angle ABC$  is  $x$  degrees, what is the value of  $x$ ?



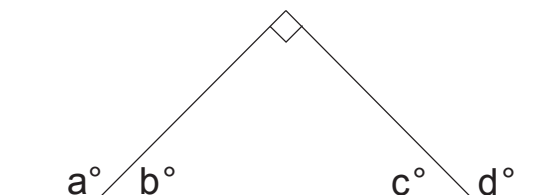
		9	2
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<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
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4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	<input checked="" type="radio"/>	9

20. In the figure below,  $a \parallel b$  and  $c \parallel d$ .  $110 < x < 130$ , what is one possible value for  $y$ ?

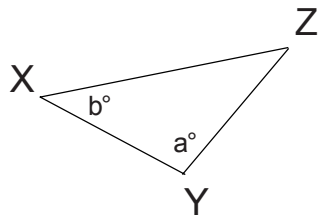


Note: Figure not drawn to scale.

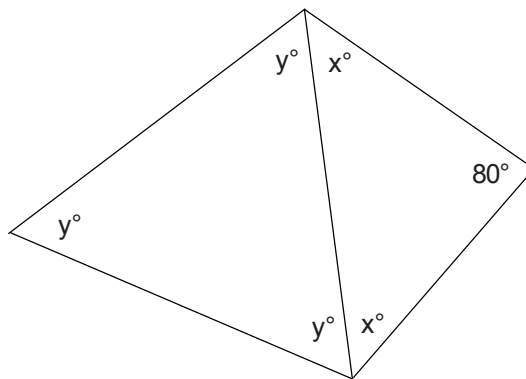
21. In the figure below, what is the value of  $(a + d) - (b + c)$ ?



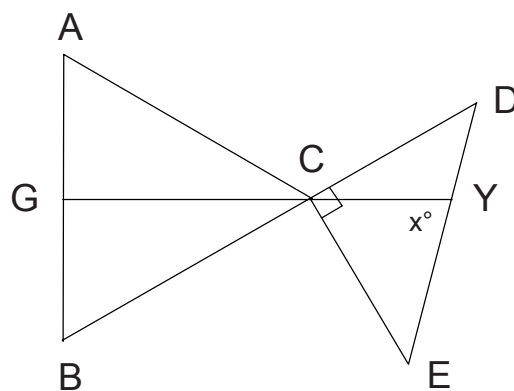
22. In the triangle below,  $XY = YZ$  and  $a = 100$ .  
What is the value of  $b$ ?



23. In the figure below, what is the value of  $x + y$ .



24. In the figure below  $G, C,$  and  $Y$  are collinear, and  $B, C,$  and  $D$  are collinear. Triangle  $ABC$  is equilateral with altitude  $\overline{GC}$ , and triangle  $CDE$  is a right isosceles triangle. What is the value of  $x$ ?



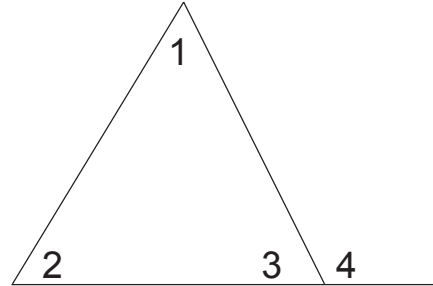
Answers: 1. B 2. E 3. D 4. C 5. E 6. D 7. C 8. D 9. D 10. B 11. D 12. E 13. E 14. C  
15. D 16. E 17. D 18. C 19. 92 20.  $50 < y < 70$  21. 180 22. 40 23. 110 24. 75

# #4

## Exterior angles of triangles

An exterior angle of a triangle is an angle that is adjacent to an interior angle.

In this figure,  $\angle 1$ ,  $\angle 2$  and  $\angle 3$  are interior angles, and  $\angle 4$  is an exterior angle.  $\angle 4$  is adjacent to  $\angle 3$ .



The measure of an exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.

Therefore, in the given figure,  $m\angle 4 = m\angle 1 + m\angle 2$ .

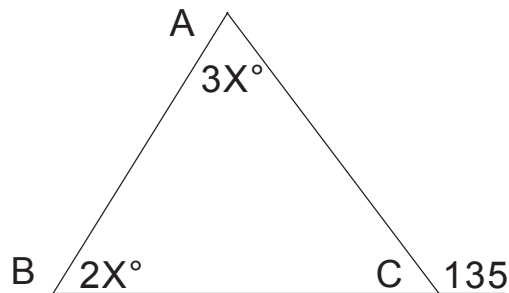
The measure of an exterior angle of a triangle is greater than the measure of either of its remote interior angles.

Therefore, in the given figure,  $m\angle 4 > m\angle 1$  and  $m\angle 4 > m\angle 2$ .

Answers are at the bottom of the next page.

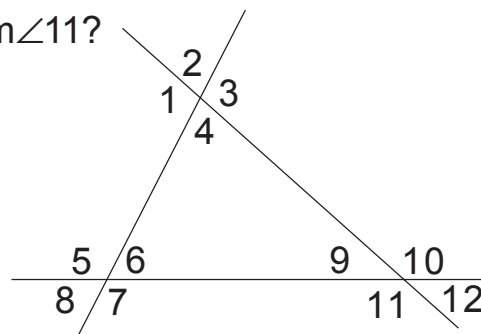
1. In the given figure, what is  $m\angle A$ ?

- (A) 27
- (B) 45
- (C) 54
- (D) 81
- (E) 90



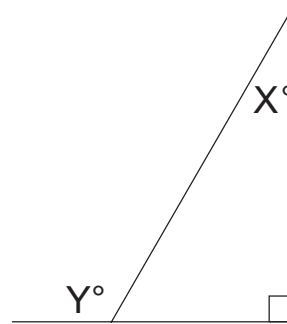
2. In the given figure, what is  $m\angle 1 + m\angle 3 + m\angle 5 + m\angle 7 + m\angle 10 + m\angle 11$ ?

- (A)  $360^\circ$  (B)  $540^\circ$  (C)  $720^\circ$
- (D)  $900^\circ$  (E)  $1080^\circ$



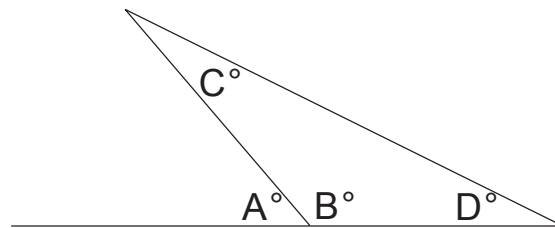
3. In the given figure, if  $Y = 4X$ , what is the value of  $Y$ ?

- (A) 30
- (B) 72
- (C) 100
- (D) 120
- (E) 135



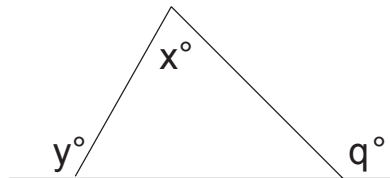
4. If  $m\angle B = 115^\circ$ , which of the following statements must be true?

- (A)  $A > C > D$
- (B)  $A > D > C$
- (C)  $A = B + C + D$
- (D)  $A > C + D$
- (E)  $B > A > C$



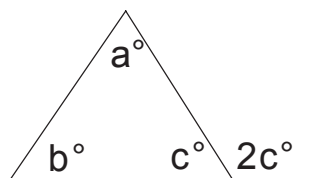
The following are student-produced response questions. You will enter your answer by marking the ovals in a grid like the one below.

5. In the figure below,  $q = 135$  and  $y = 120$ . What is the value of  $x$ ?



		7	5
<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	<input checked="" type="radio"/>
6	6	6	6
7	7	<input checked="" type="radio"/>	7
8	8	8	8
9	9	9	9

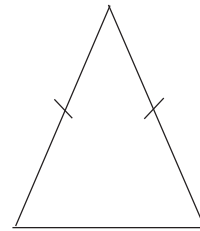
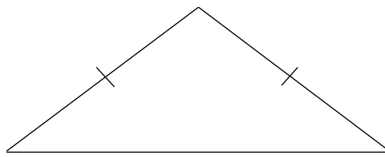
6. In the figure below, if  $a - b = 10$ , what is the value of  $a$ ?



Answers: 1. D 2. C 3. D 4. E 5. 75 6. 65

**#5**

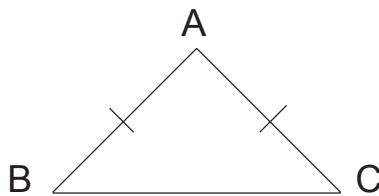
# Isosceles triangles



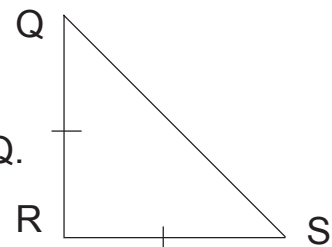
An isosceles triangle has at least two congruent sides.

If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

If  $\overline{AB} \cong \overline{AC}$   
then  $\angle C \cong \angle B$ .

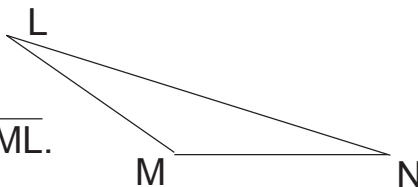


If  $\overline{QR} \cong \overline{SR}$   
then  $\angle S \cong \angle Q$ .

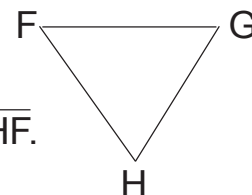


If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

If  $\angle L \cong \angle N$   
then  $\overline{MN} \cong \overline{ML}$ .

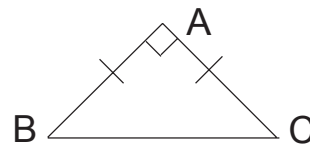


If  $\angle F \cong \angle G$   
then  $\overline{HG} \cong \overline{HF}$ .



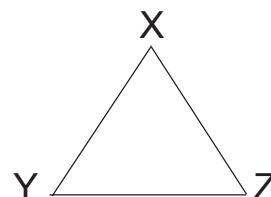
The base angles of an isosceles right triangle have measures of  $45^\circ$ .

If  $\triangle ABC$  is an isosceles right triangle,  
then  $m\angle B = 45^\circ$  and  $m\angle C = 45^\circ$ .



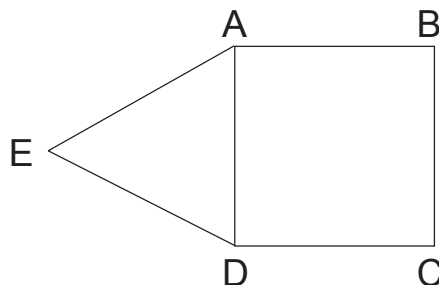
Each angle of an equilateral triangle has a measure of  $60^\circ$ .

If  $\triangle XYZ$  is an equilateral triangle,  
then  $m\angle X = 60^\circ$ ,  $m\angle Y = 60^\circ$  and  $m\angle Z = 60^\circ$ .



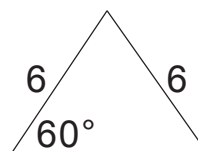
1. If ABCD is a square and  $\triangle EAD$  is an equilateral triangle, what is  $m\angle EAB$ ?

- (A)  $90^\circ$  (B)  $100^\circ$  (C)  $120^\circ$   
 (D)  $135^\circ$  (E)  $150^\circ$



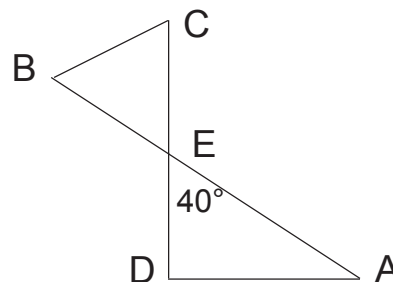
2. The perimeter of the given triangle is?

- (A)  $6\sqrt{2}$  (B)  $12 + 6\sqrt{2}$  (C) 15 (D) 18  
 (E) It cannot be determined from the information given.



3. In the given figure, if  $BE = CE$  and  $m\angle DEA = 40^\circ$ , what is  $m\angle C$ ?

- (A)  $20^\circ$  (B)  $40^\circ$  (C)  $70^\circ$   
 (D)  $80^\circ$  (E)  $140^\circ$

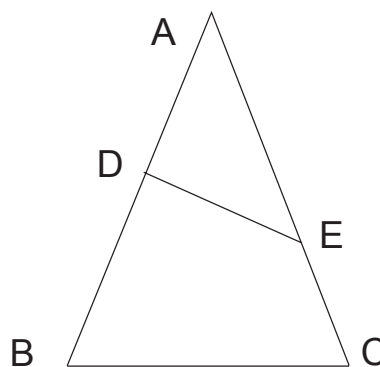


4. If  $a$ ,  $b$  and  $c$  are the degree measures of the three angles of an isosceles triangle and  $c = 120^\circ$ , then what is the value of  $b$ ?

- (A)  $20^\circ$  (B)  $30^\circ$  (C)  $60^\circ$  (D)  $90^\circ$  (E)  $120^\circ$

5. In the given figure,  $AB = AC$ ,  $m\angle AED = 30^\circ$ , and  $m\angle BDE = 80^\circ$ . What is  $m\angle B$ ?

- (A)  $45^\circ$  (B)  $55^\circ$  (C)  $60^\circ$   
 (D)  $65^\circ$  (E)  $80^\circ$



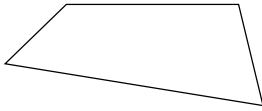
6. If  $a$ ,  $b$  and  $c$  are the degree measures of the three angles of an isosceles triangle, and  $c = 80^\circ$ , then what is the value of  $b$ ?

- (A)  $10^\circ$  (B)  $20^\circ$  (C)  $40^\circ$  (D)  $50^\circ$   
 (E) It cannot be determined from the information given.

Answers 1. E 2. D 3. C 4. B 5. D 6. E (b can be  $20^\circ$  or  $80^\circ$  as part of an  $80^\circ$ - $80^\circ$ - $20^\circ$  triangle, or b can be  $50^\circ$  as part of an  $80^\circ$ - $50^\circ$ - $50^\circ$  triangle)

#6

# Quadrilaterals



A quadrilateral is a polygon with four sides.

The sum of the measures of the four angles of a quadrilateral is  $360^\circ$ .

A special type of quadrilateral is a parallelogram.

A parallelogram is a quadrilateral in which both pairs of opposite sides are parallel.

If  $\overline{AB} \parallel \overline{DC}$  and  $\overline{AD} \parallel \overline{BC}$ , then quadrilateral ABCD is a parallelogram.



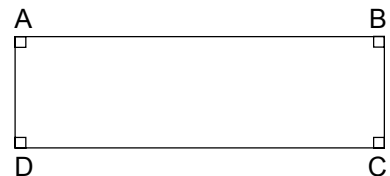
The following things are true about a parallelogram.

- |  |   |
|--|---|
| 1. Opposite sides are parallel.          | $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$ |
| 2. Opposite sides are congruent.         | $\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$         |
| 3. Opposite angles are congruent.        | $\angle A \cong \angle C$ and $\angle B \cong \angle D$                             |
| 4. Consecutive angles are supplementary. |   |

Another type of quadrilateral is a rectangle.

A rectangle is a parallelogram with four right angles.

If quadrilateral ABCD is a parallelogram, and if angles A, B, C and D are all right angles, then quadrilateral ABCD is a rectangle.



The following things are true about a rectangle.

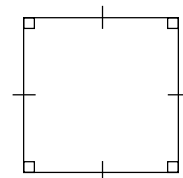
- All of the things that are true for a parallelogram.
- All four angles are right angles.
- The diagonals are congruent.

Another type of quadrilateral is a square.

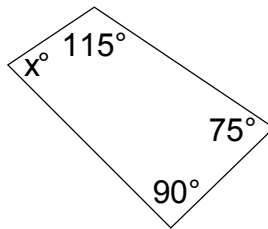
A square is a rectangle with four congruent sides.

The following things are true about a square.

- All of the things that are true for a rectangle.
- All four sides of a square are congruent.



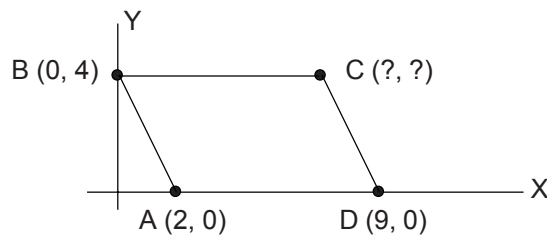
1. In the quadrilateral below, what is the value of  $x$ ?



- (A) 75 (B) 80 (C) 85 (D) 90 (E) 100

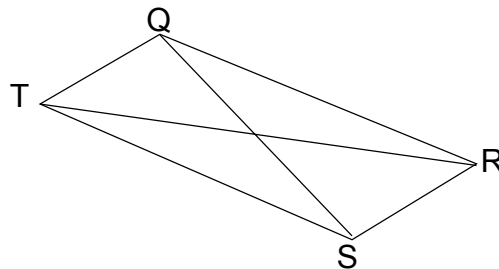
2. On the coordinate plane below, in order for quadrilateral ABCD to be a parallelogram, what must the coordinates of point C be?

- (A) (2, 4) (B) (5, 4) (C) (6, 4) (D) (7, 4) (E) (9, 4)



3. In parallelogram QRST below, if  $m\angle QRS < m\angle RST$ , which of the following must be true.

- (A)  $QR > RS$  (B)  $QS < RT$  (C)  $QS > RT$  (D)  $QS = RT$  (E)  $QR > TS$



Note: Figure not drawn to scale.

Answers: 1. B 2. D 3. B

**#7**

# Ratio and proportion

The answers are at the bottom of the last page.

1. The ratio of Joe's marbles to Bill's marbles is 3:4. The ratio of Joe's marbles to Nancy's marbles is 2:5. What is the ratio of Bill's marbles to Nancy's marbles? (Explanation below.)

(A) 2:1 (B) 3:10 (C) 1:2 (D) 4:5 (E) 8:15

---

The ratios will be  $\frac{\text{Joe}}{\text{Bill}} = \frac{3}{4}$                        $\frac{\text{Joe}}{\text{Nancy}} = \frac{2}{5}$

First, find the name that appears twice. In this problem, it is Joe. Now rewrite the ratios so that the number next to each Joe is the same. The following is one possibility.

$$\frac{\text{Joe}}{\text{Bill}} = \frac{6}{8} \qquad \frac{\text{Joe}}{\text{Nancy}} = \frac{6}{15}$$

When the common name (Joe) has the same number, any two names can be compared.

In this problem,  $\frac{\text{Bill}}{\text{Nancy}} = \frac{8}{15}$                       or 8:15

Use the same strategy for #2 below. Find the length that appears twice (AD), and write equivalent ratios that will cause the value next to each AD to be the same number.

2. Points A, B, C, and D lie on a line from left to right in that order.

If  $\frac{AB}{AD} = \frac{2}{3}$  and  $\frac{AC}{AD} = \frac{3}{4}$ , what is the value of  $\frac{AB}{AC}$  ?

(A) 2/3 (B) 4/9 (C) 1:2 (D) 8/9 (E) 9/8

- For the next question, you must know if proportions are equivalent.
- Two proportions are equivalent if you get the same result after cross multiplying in each of them.
- For example  $a/b = c/d$  is equivalent to  $a/c = b/d$  because after cross multiplying in each proportion, the result is the same,  $ad = bc$ .

3. Which of the following proportions is not equivalent to  $a/b = c/d$ .

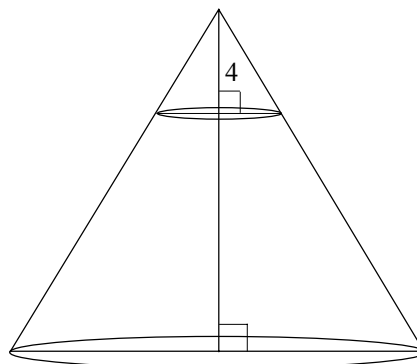
- (A)  $a/c = b/d$  (B)  $b/a = d/c$  (C)  $b/d = a/c$  (D)  $c/b = a/d$  (E)  $c/d = a/b$

The following are student-produced response questions. You will enter your answer by marking the ovals in a grid like the one to the right.

	4	/	3
<input type="radio"/>	<input checked="" type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	<input checked="" type="radio"/>
4	<input checked="" type="radio"/>	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

4. On a map,  $1/4$  inch represents 16 miles. If two towns are 160 miles apart, what is the length in inches on the map that the two towns are apart?

5. A right circular cone shown below is cut by a plane parallel to its base to form a new smaller cone that has a height of 4. If the area of the circular base of the smaller cone is  $9\pi$ , and the area of the circular base of the larger cone is  $81\pi$ , what is the height of the larger cone?

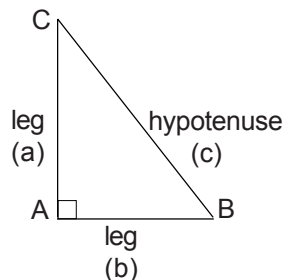
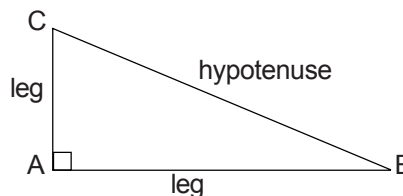


Answers: 1. E 2. D 3. D 12.  $5/2$  or 2.5 14. 12

# #8

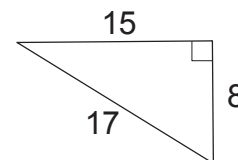
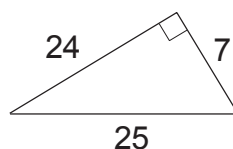
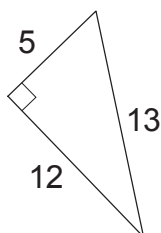
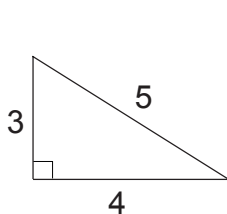
## The Pythagorean theorem and special right triangle ratios

- Right triangles, which are triangles that contain a  $90^\circ$  angle, are an important part of the PSAT and SAT.
- The longest of the three sides of a right triangle is called the hypotenuse. The other two sides are called legs.
- In right  $\triangle ABC$ ,  $\overline{BC}$  is the hypotenuse, and  $\overline{AB}$  and  $\overline{AC}$  are both legs.

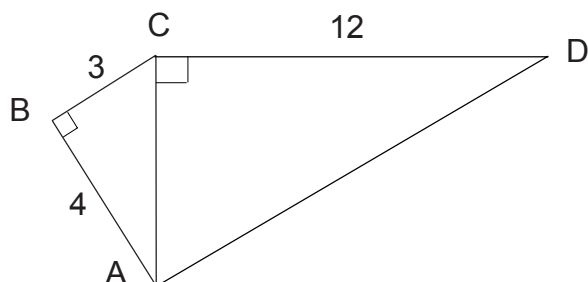


- It is important to learn the **Pythagorean theorem**,  $a^2 + b^2 = c^2$ .
- **a** and **b** represent the two legs, and **c** represents the hypotenuse.
- In other words,  $(\text{leg})^2 + (\text{leg})^2 = (\text{hypotenuse})^2$ .

- 
- There are certain relationships or ratios known as Pythagorean triples. Memorize these four and be able to apply them. 3-4-5, 5-12-13, 7-24-25, 8-15-17



- Multiples of these ratios also work. For example, some multiples of 3-4-5 are 6-8-10, 12-16-20, and 15-20-25.
- If you know the triples, you can easily find the length of  $\overline{AD}$  in the following.



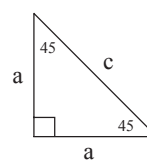
answer  
 $AD = 13$

# #8

## The Pythagorean theorem and special right triangle ratios continued

- In addition to the four Pythagorean triples, you should know about two special right triangles. These are the  $45^\circ - 45^\circ - 90^\circ$  right triangle and the  $30^\circ - 60^\circ - 90^\circ$  right triangle.

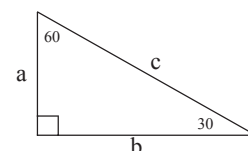
- In a  $45^\circ - 45^\circ - 90^\circ$  right triangle, which is a right isosceles triangle, the hypotenuse is equal to a leg times the square root of 2. The ratio of the three sides is  $1:1:\sqrt{2}$ .



$$c = a\sqrt{2}$$

- In the diagram to the right of a  $45^\circ - 45^\circ - 90^\circ$  right triangle, each of the legs is represented by **a** and the hypotenuse is represented by **c**.

- In a  $30^\circ - 60^\circ - 90^\circ$  right triangle, there are two relationships that are helpful to remember.
- The hypotenuse is equal to twice the smaller leg.
- The longer leg is equal to the shorter leg times the square root of 3.
- The ratio of the three sides is  $1:\sqrt{3}:2$ .

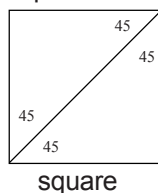
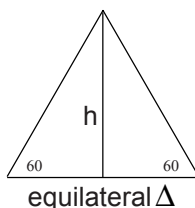


$$c = 2a$$
$$b = a\sqrt{3}$$

- In the diagram to the right of a  $30^\circ - 60^\circ - 90^\circ$  right triangle, the shorter leg is represented by **a**, the longer leg is represented by **b**, and the hypotenuse is represented by **c**.

### Two common applications of these relationships

The altitude ( $h$ ) of an equilateral creates two  $30-60-90$   $\Delta$ s. Thus, the altitude is opposite a  $60^\circ$  angle and would be equal to a side of the triangle times the square root of 3 divided by 2. So, if a side of the triangle is 8, the altitude is  $4\sqrt{3}$ .



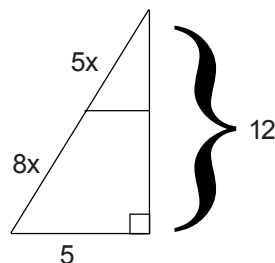
The diagonal of a square creates two  $45-45-90$   $\Delta$ s and the diagonal is the hypotenuse of each  $\Delta$ .

If a side of a square has a length of 5, then the diagonal would be  $5\sqrt{2}$ .

Answers are at the bottom of the last page.

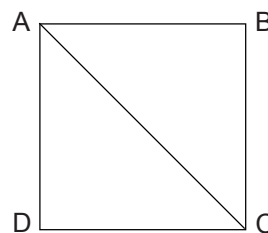
1. In the given figure, what is the value of  $x$ ?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 13



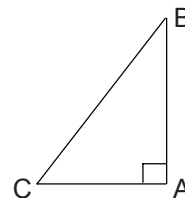
2. Quadrilateral  $ABCD$  is a square with a perimeter of 20. What is the length of diagonal  $\overline{AC}$ ?

- (A) 5 (B)  $5\sqrt{2}$  (C)  $5\sqrt{3}$  (D) 10 (E) 12.5



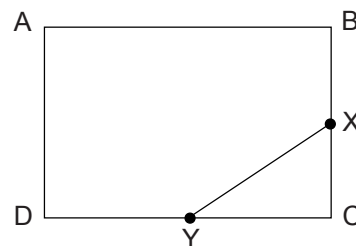
3. What is the length of  $\overline{AB}$  in the given triangle if  $CA = 2$  and  $BC = 6$ ?

- (A) 2 (B) 4 (C)  $4\sqrt{2}$  (D) 16 (E) 32



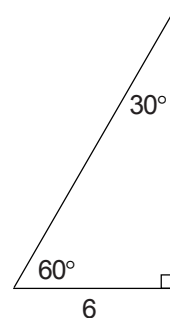
4. In the given figure, quadrilateral  $ABCD$  is a rectangle,  $X$  is the midpoint of  $\overline{BC}$ , and  $Y$  is the midpoint of  $\overline{DC}$ . If  $AB = 8$  and  $BC = 6$ , what is the perimeter of  $\triangle XYZ$ ?

- (A) 5 (B) 10 (C) 11 (D) 12 (E) 14

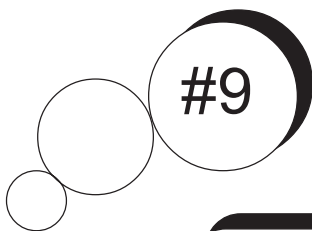


5. If the shorter leg of a 30-60-90 triangle is 6, what is the length of the longer leg?

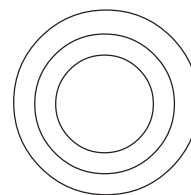
- (A) 6 (B) 8 (C)  $6\sqrt{2}$  (D)  $6\sqrt{3}$  (E) 12







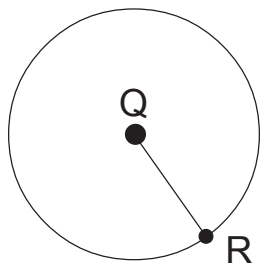
# Circles



A circle is the set of all points in a plane that are a given distance from a fixed point in the plane.

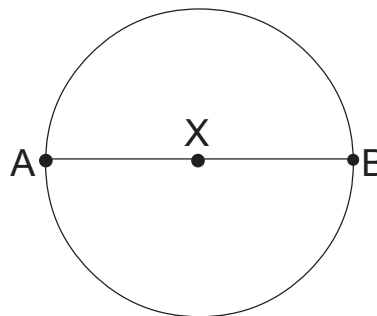
In geometry class, you learn many things about circles, but many of them do not appear or are very rare on SATs and PSATs.

The following short summary covers the main items that will be tested.



This is a diagram of circle Q. **Q** is the fixed point, and the circle consists of all the points that are a given distance from the fixed point. This distance is known as the radius and is represented by  $\overline{QR}$ .

In the circle X to the right, segment  $\overline{AB}$  is known as a diameter, because it is a chord (a segment whose endpoints lie on the circle) that passes through the center of the circle. Notice that AX and BX are radii (plural of radius).



You may have concluded correctly that

in the same circle a diameter is twice as long as a radius, or  $d = 2r$ .

**There are two important formulas to remember about a circle, and these are given in the directions at the beginning of each PSAT and SAT math section.**

Formula for circumference

$$C = \pi d \text{ or } C = 2\pi r$$

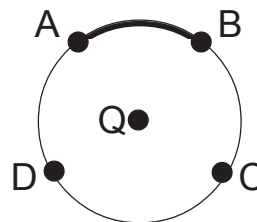
Formula for area

$$A = \pi r^2$$

Sections of a circle are called arcs.

Arc AB in circle Q is highlighted.

The following are some other arcs of the circle: arc BC, arc CD, arc DA, arc ABC, arc BCD and arc DAB.



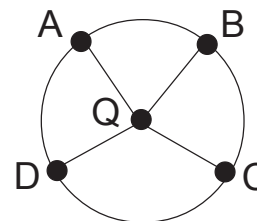
All arcs have a degree measure.

The measure of a minor arc (less than  $180^\circ$ ) is equal to the measure of its central angle.

A central angle is an angle whose vertex is the center of the circle.

$\angle AQB$  is a central angle. Arc AB will have the same measure as  $\angle AQB$ .

Some other central angles are  $\angle BQC$ ,  $\angle CQD$  and  $\angle AQD$ .



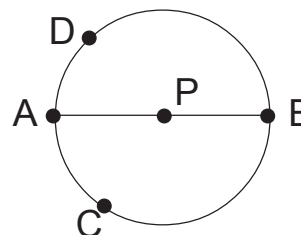
The sum of all of the nonoverlapping arcs of a circle is  $360^\circ$ . In the given diagram, this would mean:

$$m \text{ arc AB} + m \text{ arc BC} + m \text{ arc CD} + m \text{ arc DA} = 360^\circ$$

A semicircle is an arc whose endpoints are the endpoints of a diameter.

In circle P, arc ADB and arc ACB are semicircles.

The measure of a semicircle is  $180^\circ$ .



You may have to find the length of an arc.

An arc is a fractional part of the circumference of the circle. To find the fraction, put the measure of the arc over  $360^\circ$ . For instance, an arc that has a measure of  $90^\circ$  would be  $1/4$  of the circle. So, if the circle has a circumference of 24, the arc would have a length of 6,  $1/4$  of the circumference.

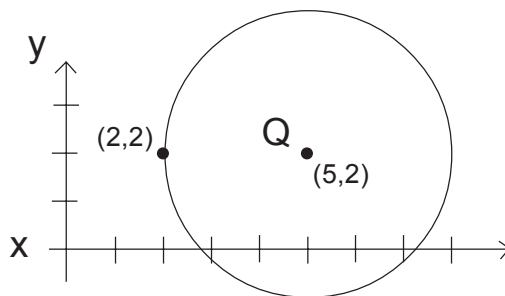
To find the length of an arc, you need to know the circumference and the measure of the arc. Then, use the following formula.

$$\frac{m \text{ arc}}{360^\circ} = \frac{\text{length of arc}}{\text{circumference}}$$

The answers are at the bottom of the last page of the section..

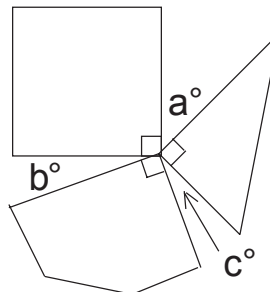
1. In the given figure, what is the circumference of the circle with center Q?

- (A)  $2\pi$  (B)  $3\pi$  (C)  $5\pi$   
 (D)  $6\pi$  (E)  $12\pi$



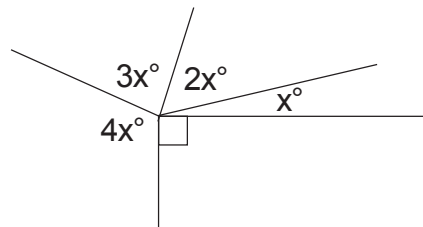
2. In the given figure, a right  $\Delta$ , a square, and a pentagon that each have a  $90^\circ$  angle intersect at the same point. What is the value of  $a + b + c$ ?

- (A) 45 (B) 60 (C) 90  
 (D) 120 (E) 270



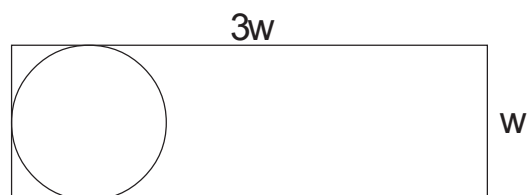
3. In the given figure, what is the value of  $x$ ?

- (A) 25 (B) 27 (C) 28  
 (D) 30 (E) 36



4. In the given figure, a circle touches three sides of a rectangle. Which of the following represents the area of the circle?

- (A)  $\frac{\pi w^2}{4}$  (B)  $\frac{\pi w^2}{2}$  (C)  $\pi w^2$  (D)  $2\pi w^2$  (E)  $4\pi w^2$



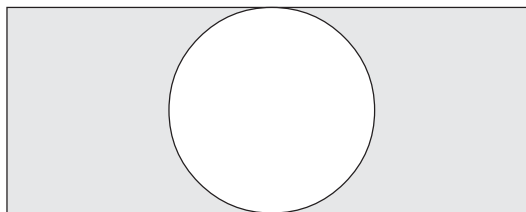
5. Two circles with radii  $r$  and  $r + 3$  have areas that differ by  $27\pi$ . What is the area of the larger circle?

- (A)  $16\pi$  (B)  $25\pi$  (C)  $36\pi$  (D)  $49\pi$  (E)  $64\pi$

6. If the radius of a unicycle's wheel is 1.5 feet, how many feet does the unicycle travel if the wheel makes three complete revolutions along a straight path?

- (A)  $1.5\pi$  (B)  $3\pi$  (C)  $4.5\pi$  (D)  $6\pi$  (E)  $9\pi$

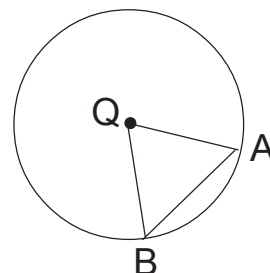
7. In the given figure, the quadrilateral is a rectangle, and the circle touches the rectangle at two points. If the length of the rectangle is 18 and the width is 6, what is the area of the shaded region?



- (A)  $48 - 9\pi$  (B)  $108 - 6\pi$  (C)  $54 - 9\pi$  (D)  $108 - 9\pi$  (E)  $108 - 36\pi$

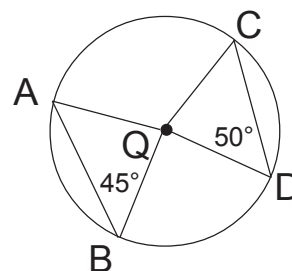
8. In the given circle that has its center at Q,  $m\angle A = 55^\circ$ . What is  $m\angle Q$ ?

- (A)  $55^\circ$  (B)  $65^\circ$  (C)  $70^\circ$   
(D)  $75^\circ$  (E)  $110^\circ$



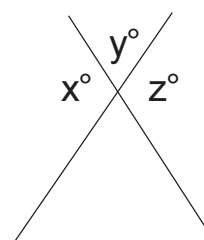
9. In the given circle that has its center at Q,  $m\angle B = 45^\circ$  and  $m\angle D = 50^\circ$ . What is  $m\angle AQC + m\angle BQD$ ?

- (A)  $95^\circ$  (B)  $170^\circ$  (C)  $180^\circ$   
(D)  $190^\circ$  (E)  $265^\circ$



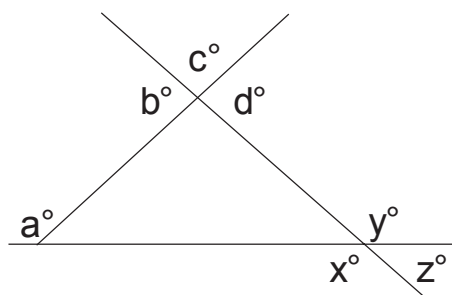
10. In the given figure, the triangle is equilateral. What is the value of  $x + y + z$ ?

- (A)  $180^\circ$  (B)  $225^\circ$  (C)  $270^\circ$   
(D)  $300^\circ$  (E)  $360^\circ$

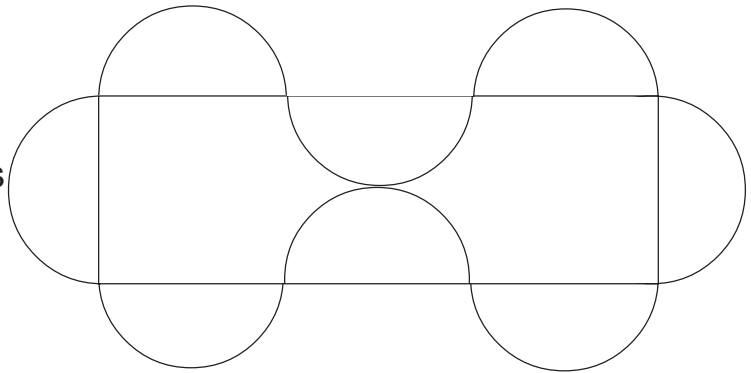


11. In the given figure, what is the value of  $a + b + c + d + x + y + z$ ?

- (A)  $360^\circ$  (B)  $540^\circ$   
(C)  $720^\circ$  (D)  $900^\circ$   
(E)  $1080^\circ$



12. In the given figure, there is a rectangle and eight semicircles of equal diameter. If the length of the rectangle is 24, what is the length of the curved path?



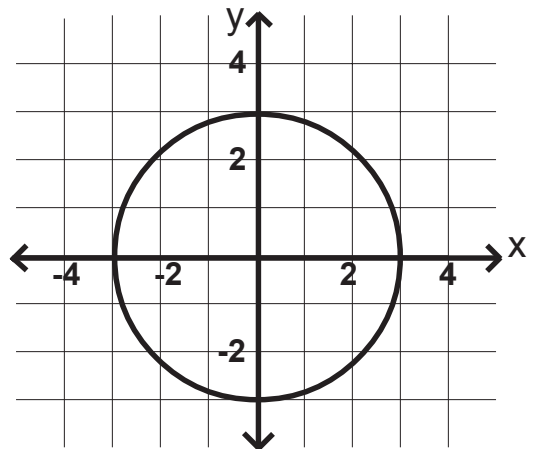
- (A)  $24\pi$  (B)  $32\pi$  (C)  $48\pi$   
 (D)  $64\pi$  (E)  $128\pi$

13. If the radius of a circle is 6, which of the following must be true about any chord of the circle that is not a diameter?

- (A) The length of the chord is equal to 6.  
 (B) The length of the chord is greater than 6.  
 (C) The length of the chord is less than 12.  
 (D) The length of the chord is equal to 12.  
 (E) The length of the chord is greater than 12.

14. In the given diagram, what is the circumference of the circle?

- (A)  $3\pi$  (B)  $6\pi$  (C)  $9\pi$   
 (D)  $12\pi$  (E)  $18\pi$



15. In the given diagram, what is the area of the circle?

- (A)  $3\pi$  (B)  $6\pi$  (C)  $9\pi$   
 (D)  $12\pi$  (E)  $18\pi$

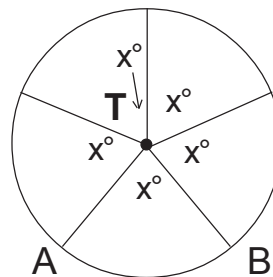
16. If the radius of a circle is tripled, what is the ratio of the area of the original circle to the area of the larger circle?

- (A) 1:1 (B) 1:3 (C)  $1:3\pi$  (D) 1:9 (E)  $\pi:9$

17. Given circle T and five central angles that each have a measure of  $x^\circ$ , what is the measure of arc AB?

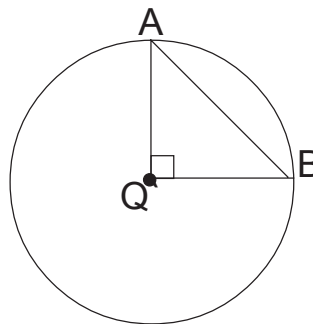
(Hint: the measure of an arc is equal to the measure of its central angle.)

- (A)  $36^\circ$  (B)  $60^\circ$  (C)  $72^\circ$   
 (D)  $90^\circ$  (E)  $144^\circ$



18. In the given figure, right  $\triangle AQB$  has a vertex at the center of circle Q. If  $AQ = 6$ , what is the length of arc AB?

- (A)  $\pi/4$  (B)  $3\pi$  (C)  $6\pi$   
 (D)  $9\pi$  (E)  $12\pi$



Recognize that arc AB is a part of the whole circumference.

If, for example, the circumference of some circle is 20 and an arc is  $1/4$  of the whole circle, then the length of the arc would be 5.

**To do #18, you must find the answer to this question.  
 The arc is what fractional part of the whole circle?**

The measures of all arcs of a circle add up to  $360^\circ$ .

So, in this problem, you find the measure of arc AB and determine what part of the whole circle ( $360^\circ$ ) it is.

The measure of an arc is equal to the measure of its central angle.

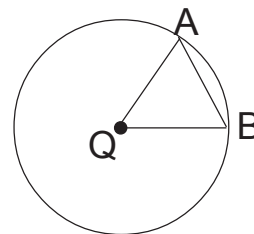
In this problem, the measure of the central angle is  $90^\circ$ , so the measure of the arc is  $90^\circ$ .

Since  $90^\circ$  (the measure of arc AB) is  $1/4$  of  $360^\circ$ , the length of arc AB is  $1/4$  of the circumference of the whole circle.

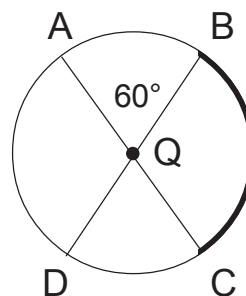
The circumference of the whole circle is found by using the formula  $C = 2\pi r$ . Since the radius in the problem is 6, the circumference is  $12\pi$  and arc AB is  $1/4$  of this or  $3\pi$ .

19. In the given figure, equilateral  $\triangle AQB$  has a vertex at the center of circle Q. If  $AQ = 12$ , what is the length of arc AB?

- (A)  $\pi/6$  (B)  $2\pi$  (C)  $4\pi$  (D)  $12\pi$  (E)  $24\pi$

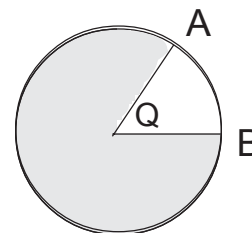


20. In the given figure, diameters  $\overline{AC}$  and  $\overline{BD}$  intersect at Q. If the radius of the circle is 15 and  $m\angle AQB = 60^\circ$ , what is the length of arc BC?



- (A)  $5\pi$  (B)  $10\pi$  (C)  $15\pi$   
 (D)  $30\pi$  (E)  $60\pi$

21. In the given figure, circle Q is composed of a shaded region and an unshaded region. If the area of the shaded region is  $80\pi$ , and the radius of the circle is 10, what is the length of arc AB?



- (A)  $4\pi$  (B)  $5\pi$  (C)  $10\pi$  (D)  $20\pi$  (e)  $100\pi$

To do this problem, you must find the circumference {fairly easy:  $C = 2\pi r$  or  $C = 2\pi(10)$ , which is  $C = 20\pi$ }.

Then, you must determine what fractional part of this circumference is arc AB. Arc AB is the same fractional part of the circumference as sector AQB is of the whole area.

The area of the whole circle is  $100\pi$  (by applying the formula  $A = \pi r^2$ ).

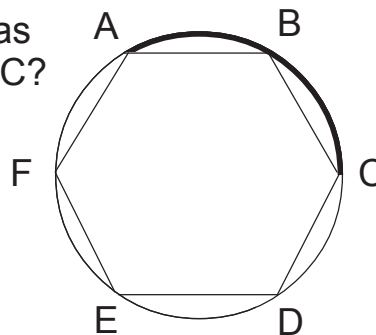
The area of the shaded region is  $80\pi$  (that was given), so the area of the unshaded region is  $20\pi$  ( $100\pi - 80\pi = 20\pi$ ).

$20\pi$  is  $1/5$  of the whole area ( $100\pi$ ). That means that arc AB is  $1/5$  of the circumference. Since the circumference is  $20\pi$ , the length of arc AB is  $4\pi$ .

22. Use the figure in problem 21 and determine the  $m\angle AQB$ .

- (A)  $36^\circ$  (B)  $45^\circ$  (C)  $60^\circ$  (D)  $72^\circ$  (E)  $80^\circ$

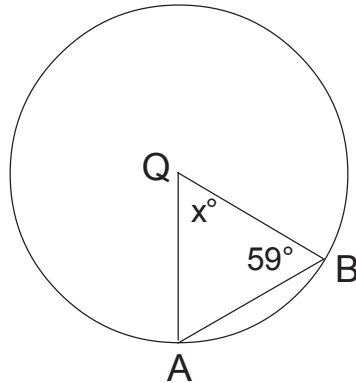
23. In the given figure, ABCDEF is a regular hexagon that is inscribed in a circle that has a radius of  $r$ . What is the length of arc ABC?



- (A)  $\frac{\pi r}{6}$  (B)  $\frac{\pi r}{3}$  (C)  $\frac{2\pi r}{3}$   
 (D)  $\pi r$  (E)  $\frac{\pi r^2}{3}$

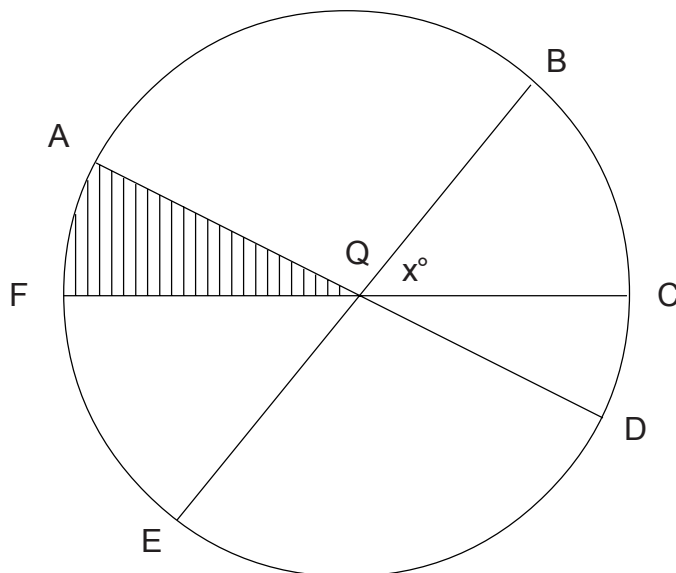
The following are student-produced response questions. You will enter your answer by marking the ovals in a grid like the one below.

24. In the given diagram, Q is the center of the circle and  $m\angle B = 59^\circ$ . The  $m\angle Q = x^\circ$ .  $\overline{QA}$  and  $\overline{QB}$  are radii. What is the value of  $x$ ?



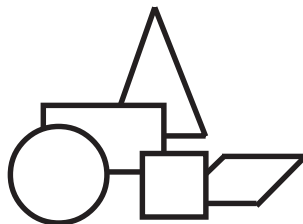
		6	2
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<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
<input type="radio"/>	0	0	0
1	1	1	1
2	2	2	<input checked="" type="radio"/>
3	3	3	3
4	4	4	4
5	5	5	5
6	6	<input checked="" type="radio"/>	6
7	7	7	7
8	8	8	8
9	9	9	9

25. In the given diagram, AD, BE, and CF are diameters, Q is the center of the circle and  $m\angle EQD = 100^\circ$ . The lined area is equal to  $1/12$  of the area of the circle. The  $m\angle BQC = x^\circ$ . What is the value of  $x$ ?

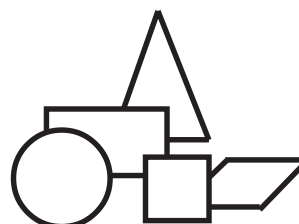


Answers: 1.D 2. C 3. B 4. A 5. C 6. E 7. D 8. C 9. D 10. D 11. C 12. B 13. C 14. B 15. C  
16. D 17. C 18. B 19. C 20. B 21. A 22. D 23. C 24. 62 25. 50

#10



## Area



Know how to find the areas of triangles, rectangles, squares, parallelograms and circles.

It is rare that you would have to find the area of any other figure on an SAT or PSAT, and when you do, the formula for that figure is given.

### Area of a triangle

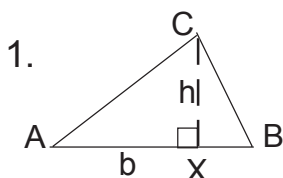
The basic formula for the area of a triangle is

$$A = \frac{1}{2}bh$$

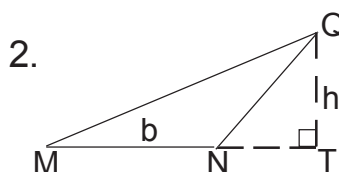
This means that the area of a triangle is equal to 1/2 of the base times the altitude.

The following are some examples:

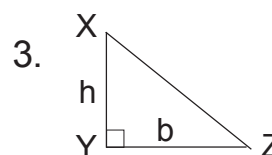
**Note:** Any side of the triangle may be the base. Each of the following examples will be done with one side designated as a base.



In  $\triangle ABC$ ,  $\overline{AB}$  is the base ( $b$ ), and  $\overline{CX}$  is the altitude ( $h$ ). If  $AB = 6$  and  $CX = 5$ , then the area of  $\triangle ABC$  would be 15 square units because  
 $A = \frac{1}{2}(6)(5) = 15$ .



In  $\triangle MNQ$ ,  $\overline{MN}$  is the base ( $b$ ), and  $\overline{QT}$  is the altitude ( $h$ ). If  $MN = 5$  and  $QT = 4$ , then the area of  $\triangle MNQ$  would be 10 square units.



In  $\triangle XYZ$ ,  $\overline{YZ}$  is the base and  $\overline{XY}$  is the altitude. If  $YZ = 16$  and  $XY = 14$ , then the area of  $\triangle XYZ$  would be 112 sq. units.

Note the third example:

If a triangle is a right triangle, then the legs will always represent the base and the altitude. Therefore, a formula to learn that applies to right triangles is the area equals 1/2 of one leg times the other leg.

For right  $\triangle$ s only:

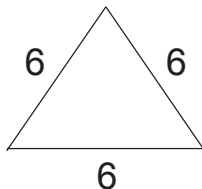
$$\text{area} = \frac{1}{2}(\text{leg})(\text{leg})$$

If a triangle is equilateral,  
there is a special formula to figure out the area.

This formula is

$$A = \frac{s^2 \sqrt{3}}{4}$$

If a side of this  
equilateral  $\Delta$  is 6,



then the area is

$$A = \frac{6^2 \sqrt{3}}{4}$$

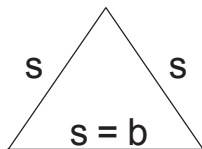
$$A = 9\sqrt{3} \text{ sq. units}$$

The following is an explanation of why this  
formula works for an equilateral triangle.  
Please ignore this if you are not interested.

The formula for the area of any triangle is

$$A = \frac{1}{2} (b)(h)$$

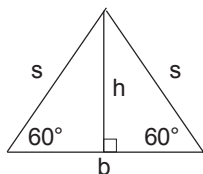
In an equilateral triangle,  
a side is the base.



Therefore, the original formula becomes

$$A = \frac{1}{2} (s)(h)$$

Notice that an altitude (h)  
in an equilateral  $\Delta$  creates  
two 30-60-90  $\Delta$ s, and the h  
is the side opposite the 60°  
angle. Using the formula  
that says that the side  
opposite the 60° angle (h in this case) in a  
30-60-90  $\Delta$  is equal to the hypotenuse (s in  
this case) times  $\sqrt{3}$  divided by 2, you find that



$$h = \frac{s\sqrt{3}}{2}$$

Now, substitute  
into the box above  
and you get

$$A = \frac{1}{2} (s) \left( \frac{s\sqrt{3}}{2} \right)$$

and after simplifying

$$A = \frac{s^2 \sqrt{3}}{4}$$

The following is a summary of the formulas for area of a triangle.

**General formula**

**For right  $\Delta$ s**

**For equilateral  $\Delta$ s**

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(\text{leg})(\text{leg})$$

$$A = \frac{s^2\sqrt{3}}{4}$$

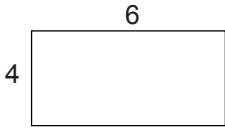
Area of a rectangle

A rectangle is a quadrilateral with four right angles.

$$A = lw$$

The area of a rectangle is the length times the width.

If the length of a rectangle is 6 and the width is 4, then the area is 24 sq. units.



$$A = (6)(4) = 24 \text{ sq. units}$$

Area of a square

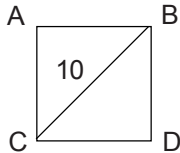
A square is a rectangle with four congruent sides. The area of a square is also the length times the width, but since the length and the width are the same, the area can be found by squaring a side.

$$A = s^2$$

If you know the length of a diagonal, there is another formula that you can use to find the area of a square. The formula is: area equals 1/2 times the square of the diagonal.

$$A = \frac{1}{2}d^2$$

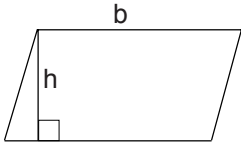
If a diagonal of square ABCD is 10, then the area is 50 sq. units.



$$A = \frac{1}{2}(10)(10) = 50$$

Area of a parallelogram

A parallelogram is a quadrilateral in which both pairs of opposite sides are parallel. Students often think of a parallelogram only as the figure to the right. However, squares and rectangles are also parallelograms.



The area of a parallelogram is the base times the altitude.

$$A = bh$$

## Area of a circle

The area of a circle is  $\pi$  times the square of the radius.

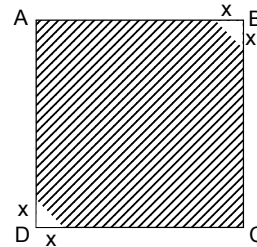
$$A = \pi r^2$$

Quite often on a PSAT or SAT, a problem appears that asks you to find the area of a shaded region.

You usually do a problem like this by finding the total area and subtracting the unshaded area.

$$\text{Shaded region} = \text{Total area} - \text{Unshaded area}$$

Example 1: Find the area of the shaded region in the given figure if ABCD is a square with each side having a length of  $9x$ .



Explanation: The area of the whole square is  $81x^2$ .

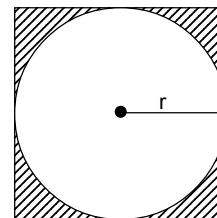
The area of each triangle is  $\frac{1}{2}(x)(x) = \frac{1}{2}x^2$ .

Therefore, the area of both triangles, which is the total unshaded area, is  $x^2$ .

$$\text{Total area} - \text{Unshaded area} = \text{Shaded region}$$

$$\text{So you have } 81x^2 - x^2 = 80x^2$$

Example 2: In the given figure, if the circle is inscribed in the square, and a side of the square is  $x$ , find the area of the shaded region in terms of  $x$ .



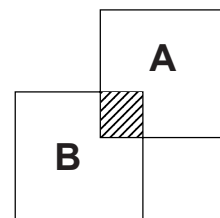
Explanation: The total area is the area of the square, which is  $x^2$ . The unshaded area is the circle, and its area is  $A = \pi r^2$ . Since the radius is  $1/2$  of a side of the square, it can be represented by  $\frac{x}{2}$ .

Substituting that into the circle formula results in

$$A = \pi \left(\frac{x}{2}\right)^2 = \frac{\pi x^2}{4} \quad \text{So the final answer is } x^2 - \frac{\pi x^2}{4}$$

Answers are at the bottom of the last page.

1. In the given figure, two squares that have sides with the same length overlap and form three regions. These are region **A**, region **B**, and the shaded region. If the area of region **A** is 7 square units and the area of the shaded region is 2 square units, what is the area, in square units, of region **B**?

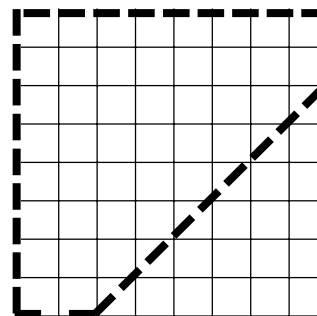


- (A) 2 (B) 5 (C) 7 (D) 9 (E) 49

2. If the area of a square is  $36x^2$ , what is the length of a side in terms of  $x$ ?

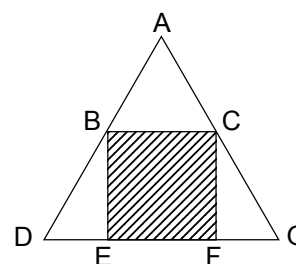
- (A)  $x$  (B)  $6x$  (C)  $18x$  (D)  $6x^2$  (E)  $9x^2$

3. The given diagram consists of squares that all have sides of  $x$  units. What is the area, in square units, of the figure that is outlined with the dotted line?



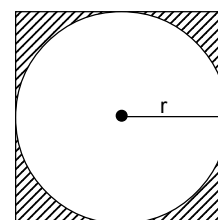
- (A)  $18x$  (B)  $18x^2$  (C)  $46x$   
(D)  $46x^2$  (E)  $64x^2$

4. In the given figure, the area of  $\triangle ADG$  is 67 square units. If the unshaded regions have a total area of 31 square units, what is the length of  $BC$ , a side of square  $BCFE$ ?



- (A) 3 (B) 6 (C) 9 (D)  $6\sqrt{2}$  (E)  $\sqrt{31}$

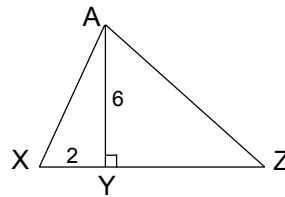
5. If a circle is inscribed in a square and the radius of the circle is  $r$ , what is the area of the shaded region in terms of  $r$ ?



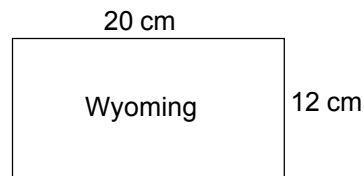
- (A)  $\pi r^2 - 8r$  (B)  $8r - \pi r^2$  (C)  $4r^2 - \pi r^2$   
(D)  $4r^2 - \pi r$  (E)  $8r - 2\pi r$

6. In the given figure,  $\triangle XZ$  has an area of 30 square units. What is the length of  $YZ$ ?

- (A) 3 (B) 6 (C) 8 (D) 10 (E) 13



7. The figure is a scale drawing of the state of Wyoming. The scale is 1 cm = 20 miles. What is the actual area, in square miles, based on this scale drawing?

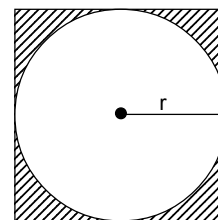


- (A) 240 (B) 1280 (C) 4800 (D) 96,000 (E) 128,000

8. If one square has a side of 2 inches, and a second square has a side of 3 inches, what, in square inches, is the sum of the areas of the two squares?

- (A) 5 (B) 6 (C) 13 (D) 20 (E) 25

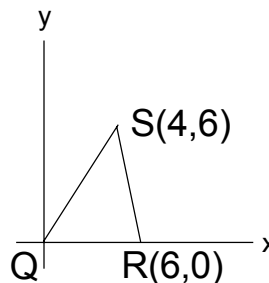
9. In the given figure, a circle is inscribed in a square. If the area of the square is 100 square units, what, in square units, is the area of the shaded region?



- (A)  $25\pi$  (B)  $100 - 25\pi$  (C)  $100 - 100\pi$   
(D) 75 (E)  $100 - 5\pi$

10. In the given figure, what is the area, in square units, of  $\triangle QRS$ ?

- (A) 12 (B) 18 (C) 24  
(D) 30 (E) 36

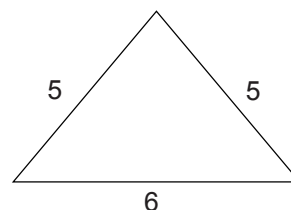


11. What is the area, in square units, of a triangle that has sides of 3, 4 and 5?

- (A) 6 (B)  $7\frac{1}{2}$  (C) 10 (D) 12 (E) 20

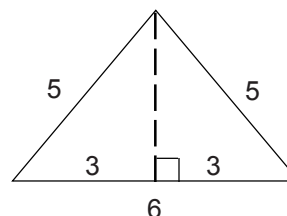
12. What is the area, in square units, of this triangle?

- (A) 12 (B)  $12\frac{1}{2}$  (C) 15 (D) 25 (E) 39



Explanation: This is not a right triangle or an equilateral triangle, so you must use the formula  $A = \frac{1}{2}bh$ . The difficulty is that you do not have an altitude in the problem, so you must draw an altitude.

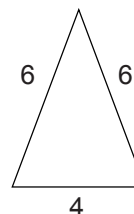
The altitude will bisect the base because the triangle is isosceles, and there are two right triangles created, each with a leg of 3 and a hypotenuse of 5. You can find the third side, which is the altitude of the big triangle, by using the Pythagorean theorem. It will be 4. From there, use the triangle area formula, and you will get 12 square units as the answer.



This is the same type of problem as 12 but a little more difficult.

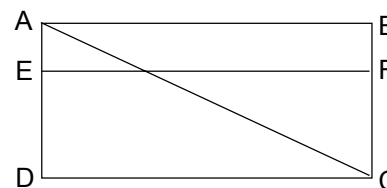
13. What is the area, in square units, of this triangle?

- (A) 12 (B)  $4\sqrt{2}$  (C)  $8\sqrt{2}$   
 (D)  $16\sqrt{2}$  (E) 64

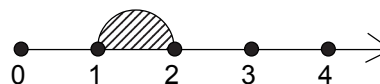


14. In the given figure, ABCD is a rectangle and  $\overline{AB} \parallel \overline{EF}$ . If  $BF = 2$ ,  $CF = 4$  and  $AC = 10$ , what is the area of quadrilateral ABFE?

- (A) 8 (B) 12 (C) 16 (D) 32 (E) 48



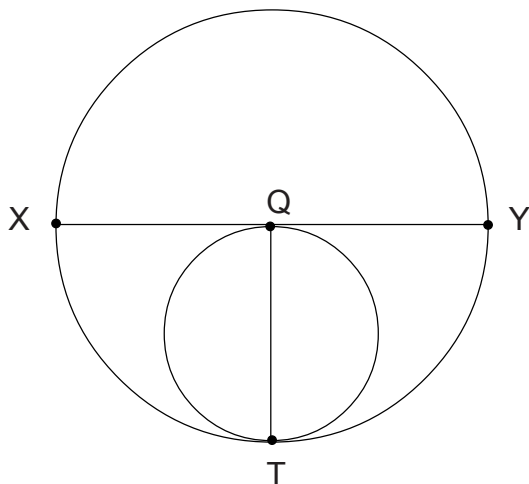
15. In the given figure, one semi-circle is drawn on the number line. What is the area, in square units, of the shaded region?



- (A)  $\pi$  (B)  $\frac{\pi}{2}$  (C)  $\frac{\pi}{4}$  (D)  $\frac{\pi}{8}$  (E)  $\frac{\pi}{16}$

The following are student-produced response questions. You will enter your answer by marking the ovals in a grid like the one below.

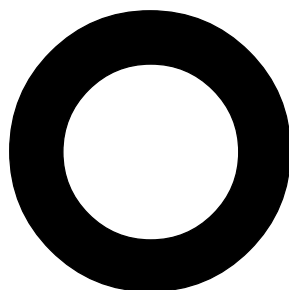
16. In the given diagram, Q is the center of the larger circle and XY is a diameter. The smaller circle is tangent to the larger circle at T and has diameter QT. If the area of the larger circle is  $100\pi$ . What is the length of the radius of the smaller circle?



			5
<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
1	0	0	0
2	1	1	1
3	2	2	2
4	3	3	3
5	4	4	4
6	5	5	5
7	6	6	6
8	7	7	7
9	8	8	8

17. The length and width of a rectangle have integer values. If the perimeter of the rectangle is 12, what is one possible value for the area of the rectangle?

18. The figure below consists of two circles that have the same center. If the shaded region has an area of  $28\pi$  and the radius of the smaller circle is 6, what is the radius of the larger circle?

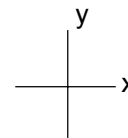


Answers: 1. C 2. B 3. D 4. B 5. C 6. C 7. D 8. C 9. B 10. B 11. A 12. A 13. C 14. C  
15. D 16. 5 17. 5, 8, or 9 18. 8

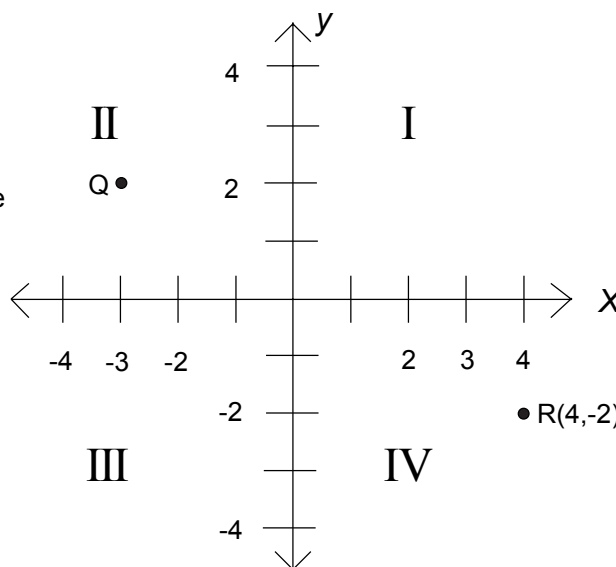
**#11**

# The coordinate plane

## (rectangular coordinate system)



- A coordinate plane is composed of a horizontal line called the x-axis and a vertical line called the y-axis.
- It has four regions that are called quadrants. These are known as quadrants I, II, III and IV.
- For every point in the coordinate plane, there is a corresponding pair of numbers. Point R has the pair (4,-2).
- 4 is the x coordinate, and -2 is the y coordinate. The coordinates of point Q in quadrant II are (-3,2).
- If you are given the coordinates of a point such as R(4,-2), you locate R by
  1. begin at the intersection of the two axes [this is called the origin and has coordinates of (0,0)]
  2. move 4 units to the right
  3. move 2 units down.

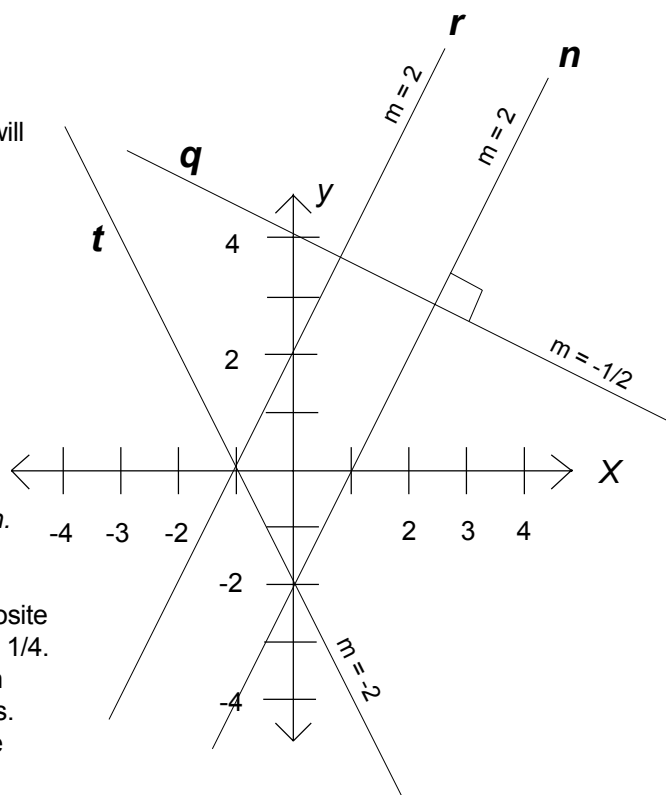


Know the following:

- Know all about slope. We expect that every test will have at least 1 question involving slope.

$$\text{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

1. As you go from left to right, a line that slants upward has a positive slope, and a line that slants downward has a negative slope. A horizontal line has a slope of 0. Vertical lines have no slope.
2. Parallel lines (as long as they are not vertical lines) have the same slope. In the diagram  $r \parallel n$ .
3. Perpendicular lines (as long as neither is a vertical line) have slopes that are opposite reciprocals. Examples of numbers that are opposite reciprocals are 2 and  $-1/2$ ,  $3/4$  and  $-4/3$ ,  $-4$  and  $1/4$ .
4. Two nonvertical lines that are reflections of each other in the x-axis or y-axis have opposite slopes. Line  $t$  is the reflection of line  $r$  in the x-axis. The slopes are opposites, 2 and  $-2$ .



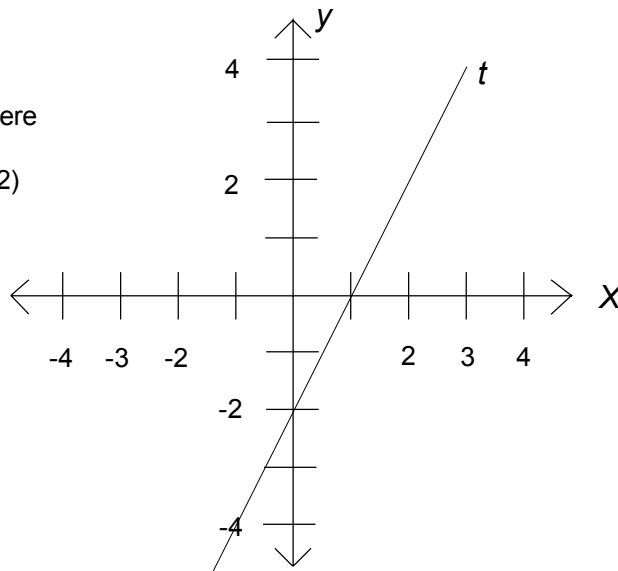
Know the following:

- Know the slope intercept form of the equation of a line. This is  $y = mx + b$ .  $m$  is the slope and  $b$  is the y-intercept. Be able to look at a graph of a line and write the equation of that line using slope and y-intercept.

**Example 1:** For the coordinate plane to the right, write the equation of line  $t$  in slope intercept form.

The y-intercept is the y-coordinate of the point where line intersects the y-axis. This is -2, so  $b = -2$ . The slope is 2, since the line passes through (0, -2) and (1, 0).

Therefore, the equation is  $y = 2x - 2$ .



**Example 2:** Line  $r$  (not shown) is perpendicular to line  $t$ . If line  $r$  passes through the point (0, 3), what is the equation of line  $r$  in slope intercept form?

Because  $r$  is perpendicular to  $t$ , and the slope of  $t$  is 2, the slope of  $r$  is  $-1/2$ . Since  $r$  passes through (0, 3),  $b = 3$ . Therefore the equation of line  $r$  is  $y = (-1/2)(x) + 3$ .

**Example 3:** Line  $r$  with equation  $y = (-1/2)(x) + 3$  crosses the x-axis at a point with coordinates  $(a, b)$ . What is the value of  $a$ ?

The y-coordinate of any point on the x-axis is 0. Therefore, the problem is asking you to find the x-value of a point that has a y-coordinate of 0. Substitute 0 for  $y$  in  $y = (-1/2)(x) + 3$  and solve for  $x$ .

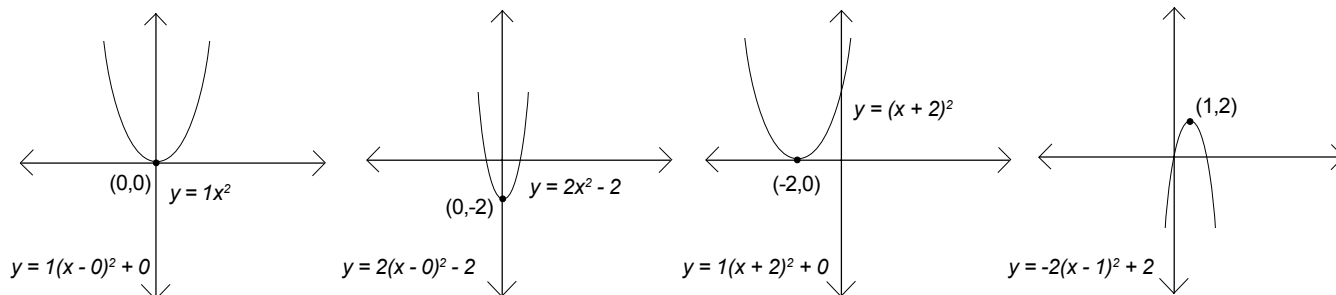
$$\begin{aligned} y &= (-1/2)(x) + 3 \\ 0 &= (-1/2)(x) + 3 \\ -3 &= (-1/2)(x) \\ 6 &= x \end{aligned}$$

Therefore, the value of  $a$  is 6.

### Parabolas

- For the SAT, know the following general parabola equation,  $y = a(x - h)^2 + k$ .
- If  $a$  is positive, the parabola opens up like a cup.
- If  $a$  is negative, the parabola opens down like a frown.
- The vertex is  $(h, k)$ .

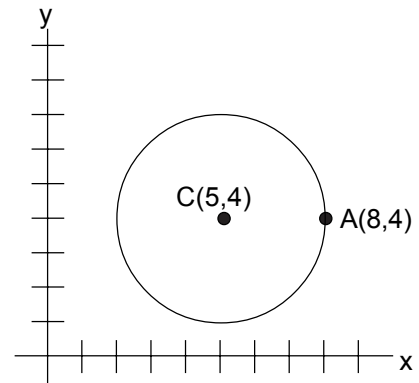
The following sketches show how the values of  $a$ ,  $h$ , and  $k$  affect the parabola. The vertex is given.



Answers are at the bottom of the last page.

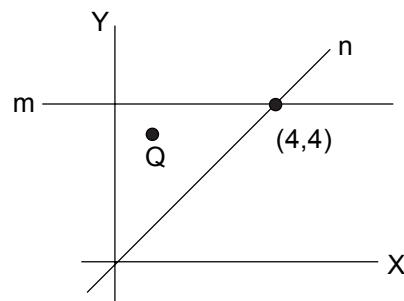
1. In the given figure, what is the circumference of the circle with center  $C$ ?

- (A)  $3\pi$  (B)  $4\pi$  (C)  $5\pi$  (D)  $6\pi$  (E)  $8\pi$



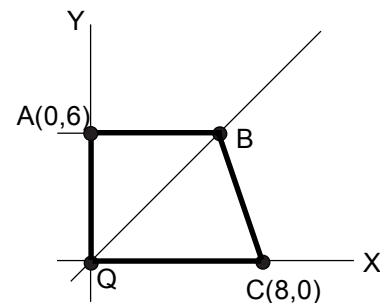
2. In the given coordinate plane, line  $m$  is parallel to the  $x$ -axis, and line  $n$  passes through the origin. Which of the following points could be the coordinates of point  $Q$ ?

- (A)  $(-1,3)$  (B)  $(1,-3)$  (C)  $(3,1)$   
 (D)  $(1,3)$  (E)  $(1,5)$



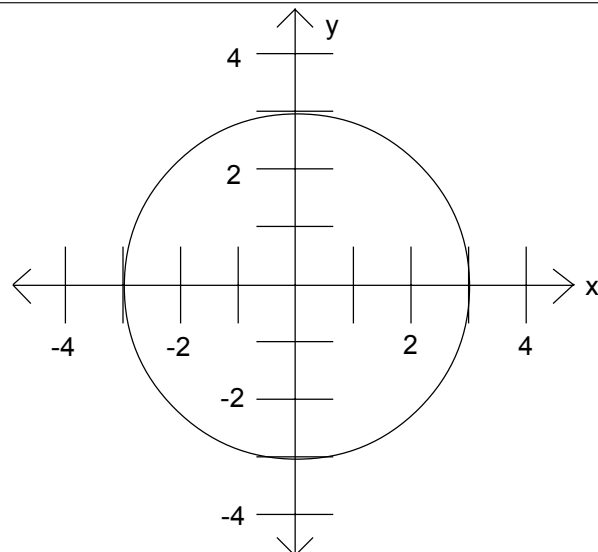
3. In the given coordinate plane,  $AB$  is parallel to the  $X$ -axis. If  $AQ = AB$ , what is the area of quadrilateral  $ABCQ$ ?

- (A) 24 (B) 28 (C) 40 (D) 42 (E) 48

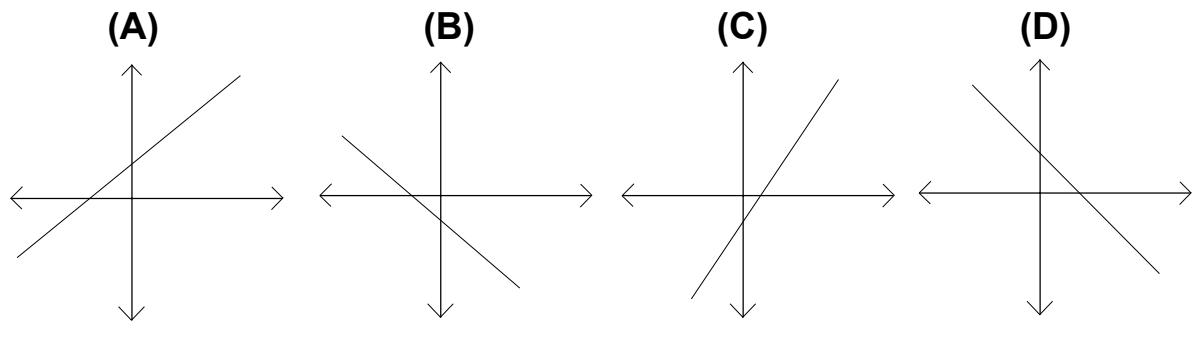


4. What is the area of the circle in the diagram. The circle's center is at the origin?

- (A)  $3\pi$   
 (B)  $6\pi$   
 (C)  $9\pi$   
 (D)  $12\pi$   
 (E)  $36\pi$

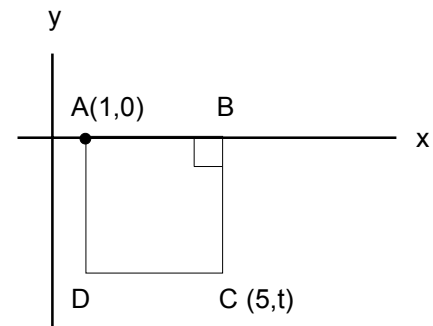


5. From the following four linear functions, which has both a negative slope and a negative y-intercept?



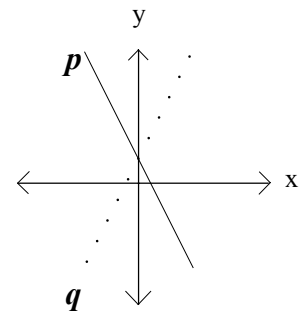
6. In the figure to the right, ABCD is a square. If the coordinates of A are  $(1,0)$ , and the coordinates of C are  $(5, t)$ , what is the value of  $t$ ?

- (A) -5 (B) -4 (C) -2 (D) 4 (E) 5



7. The equation of line  $p$  is  $y = -2x + 3$ . If the dotted line (line  $q$ ) is the reflection of line  $p$  in  $y$  (over the  $y$ -axis), what is the slope of line  $q$ ?

- (A) -3 (B) -2 (C)  $-1/2$  (D)  $1/2$  (E) 2



8. In the  $xy$ -plane, line  $r$  passes through the origin and is perpendicular to line  $t$  which  $r$  intersects at  $(4, 2)$ . What is the slope of line  $t$ .

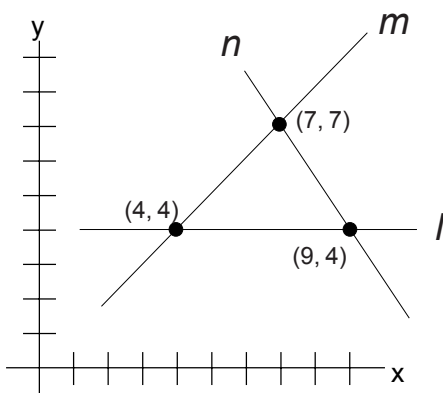
- (A) -4 (B) -2 (C)  $-1/2$  (D)  $1/2$  (E) 2

9. Using all of the same conditions in problem #8, what is the  $y$ -intercept of line  $t$ . Remember that  $t$  passes through  $(4,2)$  and from #8, you found that line  $t$ 's slope is  $-2$ .

- (A) -6 (B) 0 (C) 4 (D) 6 (E) 10

The following are student-produced response questions. You will enter your answer by marking the ovals in a grid like the one below.

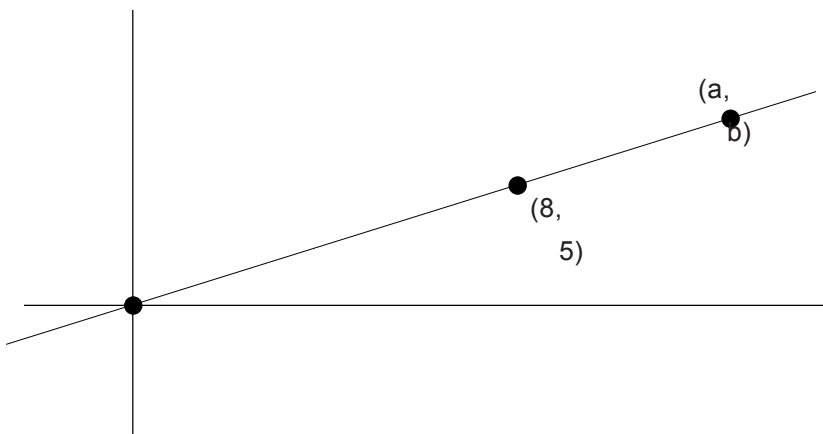
10. What is the product of the slopes of lines  $l$ ,  $m$ , and  $n$ ?



				0
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		0	0	<input checked="" type="radio"/>
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3
4	4	4	4	4
5	5	5	5	5
6	6	6	6	6
7	7	7	7	7
8	8	8	8	8
9	9	9	9	9

11. If the points  $A(4, 4)$ ,  $B(4, -3)$ , and  $C(-2, 0)$  are vertices of a triangle, what is the area of the triangle?

12. In the figure below, line  $l$  passes through the origin. What is the value of  $a/b$ ?



Answers: 1. D 2. D 3. D 4. C 5. B 6. B 7. E 8. B 9. E 10. 0 11. 21 12.  $8/5$  or  $1.6$

**#12**

# Volumes and 3-dimensional shapes

Know how to find the volume of a cube and a rectangular solid.

This is about all that you need to know concerning volume.

It is very rare that a PSAT or SAT problem would require that you know a volume formula other than these two.

If you are asked to find the volume of a different solid, you will almost always be given the formula in the problem.

## Formula for volume of a cube

$$V = e^3$$

## Formula for volume of a rectangular solid

$$V = lwh$$

You need to know all of the following concerning the cube.

1. A cube has 12 edges.
2. A cube has 6 faces.
3. The area of each face of a cube is  $e^2$ .
4. Since a cube has 6 faces, the total surface area of a cube is  $6e^2$ .
5. A diagonal of a face of a cube is  $e\sqrt{2}$ .

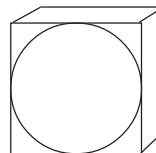
Answers are at the bottom of the next page.

1. If each edge of a rectangular solid has a dimension that is an integer greater than one, which of the following could be the volume of the solid in cubic units?

(A) 9 (B) 12 (C) 15 (D) 21 (E) 38

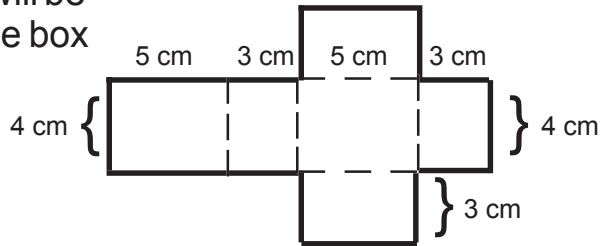
2. Cube Y has a volume of 27 cubic units. What is the length of the diameter of a circle that is inscribed in a face of the cube?

(A) 3 (B)  $3\sqrt{2}$  (C)  $3\sqrt{3}$  (D) 6 (E) 9



3. If the given figure is folded along the dashed lines, a rectangular box will be formed. What is the volume of the box in cubic centimeters?

- (A) 15 (B) 20 (C) 30  
(D) 40 (E) 60

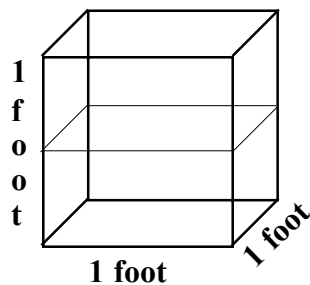


4. What is the length of an edge of a cube, in feet, that has a surface area of 150 square feet?

- (A) 2 (B) 5 (C) 10 (D) 20 (E) 25

The following are student-produced response questions. You will enter your answer by marking the ovals in a grid like the one below.

5. The container pictured below is in the shape of a cube. The cube is half full of water, and each edge of the cube has a length of 1 foot. What volume, in cubic inches, of water must be added to the container to make the level of water rise exactly one inch.



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1	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
2	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
3	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
4	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
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8	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
9	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

6. How many cubes with dimensions of 2 inches by 2 inches by 2 inches will it take to make a cube that has dimensions of 10 inches by 10 inches by 10 inches.

Answers 1. B 2. A 3. E 4. B 5. 144 6. 125

